Algebra Basics

Curriculum Ready

ACMNA: 133, 175, 176, 177, 179

www.mathletics.com

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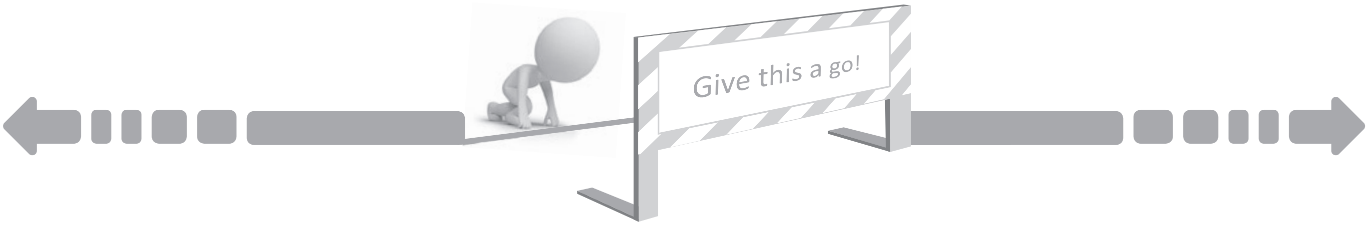
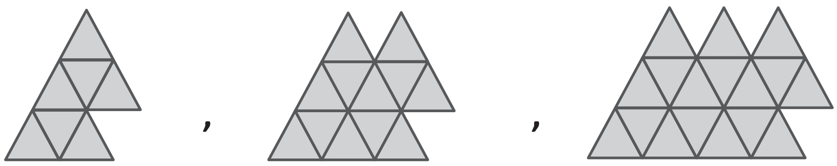
Fill in the spaces with anything you already know about Algebra

Career Opportunities:

Architects, electricians, plumbers, etc. use it to do important calculations.



Q Triangles have been stacked to form an increasing number pattern below:



7 triangles , 12 triangles , 17 triangles ,...

How many small triangles would be needed to make the 15^{th} picture in this pattern?

Work through the book for a great way to solve this

Words and symbols

Algebra uses letters or symbols called **variables**. Each part in an algebraic expression is called a **term**.

Look at the algebraic expression $4 - a$

$$4 - a$$

Constant term (Can't change) — Variable term (Can change)

if a is 1, the outcome is 3
If a is 8, the outcome is -4

Let's look at another similar expression.

$m + 9$

$$m + 9$$

Variable term (Can change) — Constant term (Can't change)

If m is 3, the outcome is 12
If m is -4, the outcome is 5

Algebraic expressions with an equals sign '$=$' are called equations.

$4 + x = 7$

$$4 + x = 7$$

The equal sign means $4 + x$ has the same value as 7

Constant — Variable — Constant

To make this correct, x must be 3

Here's another one.

$m - 5 = 9$

$$m - 5 = 9$$

The equal sign means $m - 5$ has the same value as 9

Variable — Constant — Constant

To make this correct, m must be 14

Words and symbols

WORDS AND SYMBOLS
..../...../20...

1. Write down the variable in each of the following mathematical statements:

a. $12 + b$

b. $3 - m + 2$

c. $7 \times k + 3$

d. $2a + 3a$

2. Circle each of the algebraic expressions below in which the variable can be **any** value:

$5 \times w = 30$ $\quad$ $x \div x = 1$ $\quad$ $32 - 2 \times d = 16$

$3 + x =$ $\quad$ $12 \times g =$

$200 \div s = 25$ $\quad$ $3 \times x + 6 =$

3. Match up each of the algebraic expressions with the correct outcome using the given variable value:

$11 - x$ if $x = 4$ •	• 20
$4 \times m$ if $m = 5$ •	• 10
$27 \div a$ if $a = 3$ •	• 9
$1 + 3 \times z$ if $z = 3$ •	• 7

4. Write down the value of the variable that makes these equations equal on both sides:

a. $12 + c = 20$

$c =$

b. $14 - h = 2$

$h =$

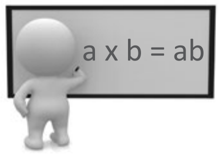
c. $k \div 3 = 6$

$k =$

d. $12 \times y = 72$

$y =$

Multiplication



Instead of writing $5 \times m$ or $a \times b$, we simply write $5m$ or ab to mean the exact same thing!

Always put the number first.

Simplify $3 \times 2 \times n$

$$3 \times 2 \times n = 6 \times n \quad \text{Multiply the numbers together}$$
$$= 6n$$

If multiplying by 1, do not write 1 in front of the variable.

Simplify $1 \times y$

$$1 \times y = y \quad (\text{not } 1y) \quad \text{The 1 is hidden}$$

1×2 is just 2. The same rule applies when multiplying a variable by 1

Write multiplied variables in alphabetical order.

Simplify $2 \times p \times 5 \times r \times q$

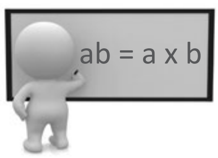
$$2 \times p \times 5 \times r \times q = 2 \times 5 \times p \times r \times q \quad \text{Re-order with numbers first}$$
$$= 10 \times p \times r \times q \quad \text{Multiply the numbers first}$$
$$= 10pqr \quad \text{Put variables in alphabetical order}$$

Use powers to simplify multiplications of the same variable.

Simplify $a \times a \times b$

$$a \times a \times b = a^2 \times b$$
$$= a^2 b$$

Just like 4×4 is 4^2, $a \times a$ is a^2



Doing the opposite of these examples is called **expanding**.

Write $a^2 b$ in expanded form

$$\therefore a^2 b = a \times a \times b$$

Simplified form

Expanded form

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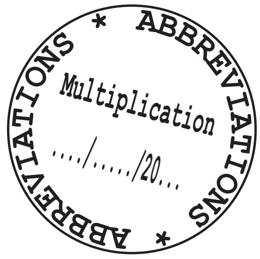
Multiplication



1 Simplify: (psst: remember the rules!)

a) $2 \times 7 \times k$

b) $u \times 1$

c) $5 \times r \times p$

d) $n \times m \times m$

e) $6 \times b \times 3 \times b$

f) $4 \times j \times l \times 3 \times k$

2 Expand each of these

a) $4pq$

b) $4a^2$

c) $3m^2n$

3 It's combo time! Calculate the value of these expressions using the variable in the square brackets.

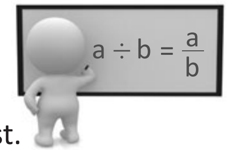
a) $3x + 2 \qquad [x = 4]$

b) $15 - 2b \qquad [b = 6]$

c) $3 \times 5g \qquad [g = 2]$

d) $4m^2 \qquad [m = 3]$

Division



When dividing two algebraic terms it sometimes helps to write the division as a fraction first.

Simplify $h \div 8$

$$h \div 8$$

Numerator — Denominator

$$\therefore h \div 8 = \frac{h}{8}$$

Aways write fractions in simplest form.

Simplify $4x \div 12$

$$4x \div 12$$

Numerator — Denominator

$$\therefore 4x \div 12 = \frac{4x}{12}$$

$$= \frac{x}{3}$$

$\frac{4}{12} = \frac{1}{3}$ when simplified

Brackets are not necessary for simple divisions written in fraction form.

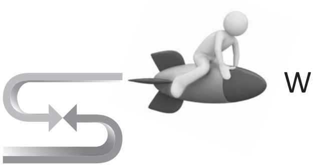
Simplify $(3 + m) \div n$

$$\underbrace{(3 + m)} \div n$$

Numerator — Denominator

$$\therefore (3 + m) \div n = \frac{3 + m}{n}$$

brackets are hidden in fraction form



When doing the **reverse** and there is more than one term, brackets must be **put in**.

$$\therefore \frac{y}{4 + x} = y \div (4 + x)$$

brackets or (parentheses)

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Division

1 Simplify by writing without using a division sign:

a $2 \div d$

b $a \div c$

c $5 \div (r + 3)$

d $(y + z) \div z$

2 Re-write these expressions using a division sign: (psst: some may need brackets)

a $\dfrac{w}{4}$

b $\dfrac{c}{3 + a}$

c $\dfrac{6}{3x + 2}$

d $\dfrac{x - y}{v + w}$

3 Re-write these expressions using a division sign: (psst: simplify the fractions first)

a $\dfrac{2a}{6}$

b $\dfrac{6b}{12c}$

c $\dfrac{15x}{20y}$

d $\dfrac{4(m + n)}{12p}$

Mixed simplifying concepts

It's combo time!

1. Simplify these by writing without multiplication or division signs:

   a. $5 \times a \div 4$

   b. $1 \times m \div (4 + n)$

   c. $n \times m \div (b \times a \times c)$

   d. $(8 \times 2p) \div (3 \times 3q)$

   e. $x \times x \div (y + 2x)$

   f. $d \times f \times d \div (11 + f \times e)$

2. Expand these by re-writing with multiplication/division signs and grouping symbols:

   a. $\dfrac{2d}{3}$

   b. $\dfrac{a + 4}{b}$

   c. $\dfrac{q - r}{9q}$

   d. $\dfrac{l^2}{j - k}$

   e. $\dfrac{5b^2}{a^2 + 2b}$

   f. $\dfrac{7xyz}{x + 7y}$

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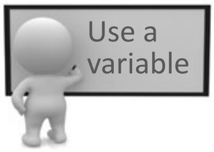
Phrases as algebraic expressions



To solve problems with algebra we use **variables** to turn the problem into an algebraic rule (or relationship).

Write a rule for: the sum of a number and 5

plus

- Give 'the number' a variable.

 Let the number be n

$\therefore$ The sum of a number and 5 is: $n + 5$

Write a rule for: the difference between a number and 3

minus

Let the number be n

$\therefore n - 3$

The order of the words in a sentence makes a difference to which operation is done first.

Write a rule for: the difference between double a number and 3

minus multiply by 2

Let the number be n

$\therefore 2n - 3$

$\therefore$ a number doubled, minus 3

Write a rule for: double the difference between a number and 3

multiply by 2 minus

Let the number be n

$\therefore 2(n - 3)$ Brackets used because $n - 3$ is calculated first

$\therefore$ double the difference between a number and 3

Write a rule for: the quotient of 4 times a number and 3

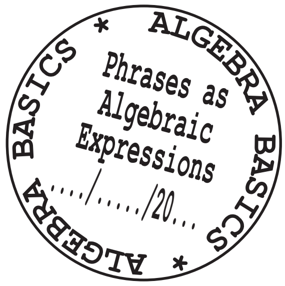
divide

$\frac{\text{numerator}}{\text{denominator}}$

Let the number be n

$\therefore \frac{4n}{3}$ $4n$ was first in the sentence, so it is the numerator

Phrases as algebraic expressions



1 Write these phrases as algebraic expressions (let the number be 'n')

a. The sum of a number and 7:

$$n + 7$$

b. The difference between 9 and a number:

c. The sum of 6 times a number and 1:

d. The product ($\times$) of a number and 4:

e. The quotient ($\div$) of two more than a number and 3:

f. The difference between a number squared and 6:

g. The product of a number minus 5 and 2:

h. 8 less than twice a number:

i. 10 added to a number halved:

j. A number multiplied by 5 more than itself:

Phrases as algebraic expressions

2 Circle whether the algebraic expression is correct or incorrect for each phrase.

a A number multiplied by 4 added to 7: Correct Incorrect

$$4n + 7$$

b The difference between a number and 4: Correct Incorrect

$$4 - n$$

c The sum of 6 and the product of 3 and a number: Correct Incorrect

$$3n + 6$$

d The quotient of 4 plus a number and 9: Correct Incorrect

$$4 \div (n + 9)$$

e A number divided by 5 and added to the number: Correct Incorrect

$$\frac{n}{5} + 5$$

f A number times the difference between the number and one: Correct Incorrect

$$n(n - 1)$$

g The sum of a number, and three minus the number halved: Correct Incorrect

$$n + \frac{3 - n}{2}$$

h The product of 6 more than twice a number and 4: Correct Incorrect

$$4(2n + 6)$$

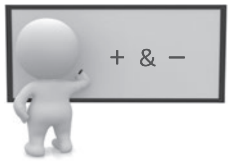
i The product of a number squared and 3: Correct Incorrect

$$(3n)^2$$

j The quotient of 5 less than a number and the number: Correct Incorrect

$$\frac{n - 5}{n}$$

Addition and subtraction



If the variable parts are **exactly** the same, the terms are called 'like terms'.

Like terms: x and $-x$ — Like terms; $3b$ and b — Like terms; $2y$ and $-5y$ — Like terms

Not Like terms: a and b — Not like terms; p and p^2 — Not like terms; $2x$ and $-5y$ — Not like terms

Only 'like terms' can be added or subtracted.

Simplify $2a + a$

$$2a + a$$

Variable parts are the same (like terms)

$$\therefore 2a + a = 3a$$

Simplify $8x - 3x$

$$8x - 3x$$

like terms

$$\therefore 8x - 3x = 5x$$

Simplify $3d + 4d + 6c$

$$3d + 4d + 6c$$

like terms

$$3d + 4d + 6c = 7d + 6c$$

This cannot be simplified any further

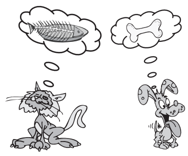
Why don't we add or subtract unlike terms? Good Question!

Let's look at a problem the last example could represent.

At a picnic for pets, each dog gets 7 treats and each cat gets 6 treats. Number of treats needed is: (7 treats × number of dogs) + (6 treats × number of cats)

$$= (7 \times d) + (6 \times c)$$

the number of dogs — the number of cats



Simplified: $= 7d + 6c$

d and c represent two different animals so it does not make sense to add them together.

Therefore $7d + 6c$ is the **simplest expression** for this problem.

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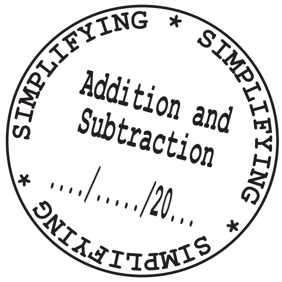
Addition and subtraction



1 Simplify:

a $a + 9a$

b $3u + 5u$

c $14r - 9r$

d $4g - 7g$

e $6m - 8m$

f $-11x + 2x$

g $7y + 2y + 4y$

h $30p - 15p - 10p$

2 Simplify: (psst: look for the like terms!)

a $13m + 9n + 12m$

b $14a + b + 10b$

c $16x + 9y + 15y$

d $9d - 5c - 3d$

e $7e + 11e + 2a$

f $13g - 15g - 4h$

Grouping like terms

Same character but not alike

Terms can have the same variable letter but still not be ‘like terms’.

Simplify $7a + 3a^2 + a + 2a^2$

- a is different to a^2 so they are not like terms.

$$\therefore\ \underbrace{7a + 3a^2 + a}_{\text{Like terms}} + 2a^2 = 7a + a + 3a^2 + 2a^2$$

Like terms: $7a$ and a; Like terms: $3a^2$ and $2a^2$

Grouping the like terms

$$= 8a + 5a^2$$

Each term of an expression includes the sign in front of it.

Simplify $9j - 11k + 5j + 8k$

$$9j - 11k + 5j + 8k = \boxed{9j}\ -11k\ \boxed{+5j}\ +8k$$

no sign in front mean +

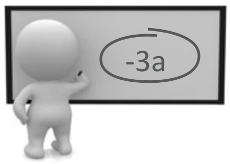
$$= \boxed{9j + 5j}\ -11k + 8k$$

Grouping the like terms

$$= 14j - 3k$$

Simplify

It’s helpful to circle the like terms with similar shapes, including the sign in front.



Here are two more examples that combine two simplifying concepts.

Simplify and write in fraction form: $(5a + 4b - 2a) \div 3b$

$$(5a + 4b - 2a) \div 3b = (3a + 4b) \div 3b$$

Simplify the bracket

$$= \frac{3a + 4b}{3b}$$

Write division as a fraction

Simplify each bracket and write in fraction form: $(x - 2x^2 + 8x) \div (13x^2 + 8x^2 - 5x)$

$$(x - 2x^2 + 8x) \div (\boxed{13x^2}\ \boxed{+8x^2} - 5x) = (9x - 2x^2) \div (21x^2 - 5x)$$

Simplify the bracket

$$= \frac{9x - 2x^2}{21x^2 - 5x}$$

Write division as a fraction

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Grouping like terms

1 Simplify: (psst: look for the like terms!)

a $9a + 3b + a + 4b$

b $4p^2 + 3p + 19p + 7p^2$

c $n - 11m - n - 12m$

d $3y - 5x + y - 8x$

e $9p - 4q + 3p + 12q$

f $14a^2 + 4b - 3a + 2a^2$

Combo time!

2 Simplify and write in fraction form:

a $11y \div (2y + 2x - y)$

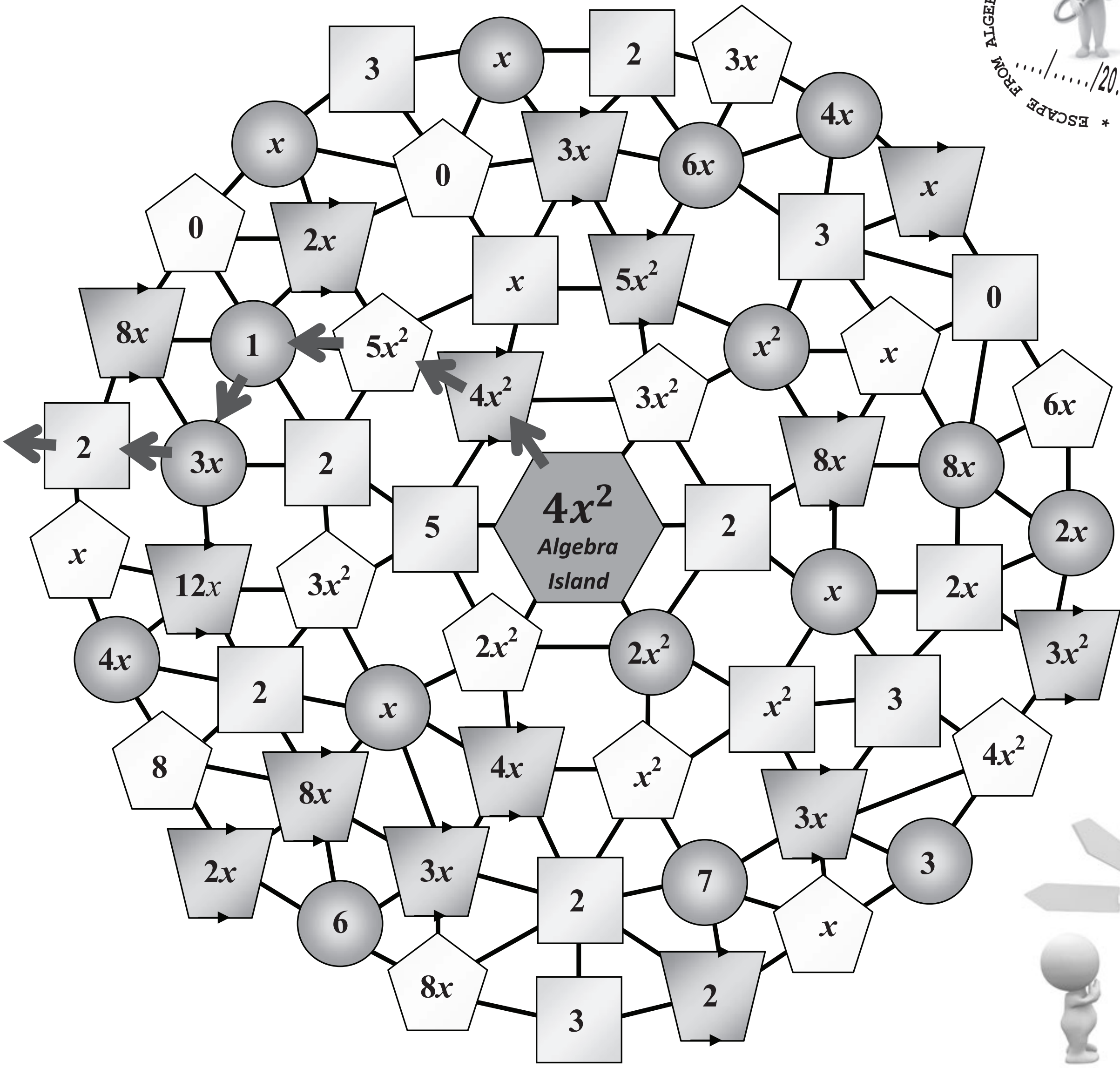
b $(7p^2 - 5p - 8p^2) \div 12$

3 Simplify each bracket and write in fraction form:

a $(2x - 3y + 2x) \div (4x + 3x - 2y)$

b $(2 \times 4a + 3 \times 2b) \div (3 \times a \times a + 2a^2)$

Escape from algebra island puzzle



Escape from Algebra Island puzzle

One path has been found for you!
$[(([(4x^2 + 4x^2) - 5x^2] \div 1) \div 3x] \times 2 = 2x$

Square steps = multiply
Circle steps = divide
Trapezium steps = add
Pentagon steps = subtract

} Remember, like terms only!

Starting with a value of $4x^2$, travel along the lines from step to step until you get to the outer edge.
Each step affects your value.
If you have exactly $2x$ left when you reach one of the shapes at the outer edge, then you have escaped!
Good luck.

How many paths can you find to get away from Algebra Island?

How many steps is the longest path you can find?

Bringing all the previous concepts together

These examples combine the different simplifying concepts together.

Calculate the value of $4a \div 3$ when $a = 6$

- When $a = 6$, $4a = 4 \times 6$, not 46!

$$4a \div 3 = 4 \times 6 \div 3$$

$a = 6$

$$= 24 \div 3$$
$$= 8$$

or

$$4a \div 3 = \frac{4a}{3}$$

$a = 6$

$$= \frac{4 \times 6}{3}$$
$$= \frac{24}{3}$$
$$= 8$$

This is useful in questions with multiple variables.

The fancy name given to doing this sort of thing in Mathematics is **substitution**.

Calculate the value of $5x + 2y$ when $x = 2$ and $y = 6$

$$5x + 2y = 5 \times 2 + 2 \times 6$$ Substitute the value of the variables.

$x = 2$ $y = 6$

$$= 10 + 12$$
$$= 22$$

Where possible, simplify the expression first before substituting in the variable values.

Evaluate $4m + 3n - 2m + 5n$ when $m = 6$ and $n = -3$

- Simplify: $4m \boxed{+ 3n} - 2m \boxed{+ 5n}$ Identify the like terms and their sign

$$= 4m - 2m \boxed{+ 3n + 5n}$$ Group the like terms

$$= 2m + 8n$$ Simplify

- Evaluate: $2m + 8n = 2 \times 6 + 8 \times -3$ Substitute in the variable values

"find the value of"

$m = 6$ $n = -3$

$$= 12 - 24$$
$$= -12$$

The same variable value can be substituted into unlike terms.

Evaluate $3p^2 + 8p - p^2 - 3p$ when $p = 2$

- Simplify:

$$3p^2 \boxed{+\,8p} - p^2 \boxed{-\,3p}$$ Identify the like terms and their sign

$$= 3p^2 - p^2 \boxed{+\,8p - 3p}$$ Group the like terms

$$= 2p^2 + 5p$$ Simplify by combining the like terms

- Evaluate:

$$2p^2 + 5p = 2 \times \underset{p\,=\,2}{2}^2 + 5 \times \underset{p\,=\,2}{2}$$ Substitute in the variable value

$$= 2 \times 4 + 5 \times 2$$
$$= 8 + 10$$
$$= 18$$

Checkout these two extra examples...

Evaluate $\dfrac{2x + y}{3x}$ when $x = 3$ and $y = 12$

Remember:
$\dfrac{\text{numerator}}{\text{denominator}}$

$$\frac{2x + y}{3x} = \frac{2 \times \overset{x\,=\,3}{3} + \overset{y\,=\,12}{12}}{3 \times \underset{x\,=\,3}{3}}$$ Substitute in the variable values

$$= \frac{6 + 12}{9}$$ Simplify the numerator and denominator

$$= \frac{18}{9} \quad (18 \div 9)$$ Simplify the fraction

$$= 2$$

Evaluate $2m^2n$ when $m = 2$ and $n = 7$

- Evaluate:

$$2m^2n = 2 \times m^2 \times n$$ Expanded form

$$2 \times m^2 \times n = 2 \times \underset{m\,=\,2}{2}^2 \times \underset{n\,=\,7}{7}$$ Substitute in the variable values

$$= 2 \times 4 \times 7$$ Multiply terms together

$$= 56$$

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Bringing all the previous concepts together

1 Calculate the value of these expressions when $v = 4$

a $4v + 2$

b $24 \div 2v$

c $10 - \frac{v}{4}$

d $\frac{2v + 6}{7}$

2 Calculate the value of these expressions when $a = -2$ and $b = 5$

a $a + 2b$

b $3b - 6a$

c $\frac{24}{a + b}$

d $\frac{a^2 b}{4}$

3 Evaluate these expressions when $c = 6$, $d = 9$

a $c + d + 2c + 3d$

b $2c + d + 3c - d$

c $\frac{2d - c}{d - c}$

d $(c + d) \times (2c - d)$

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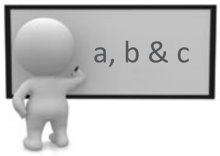


Bringing all the previous concepts together

Give these three variable questions a go!

4 Evaluate these expressions when $x = 6$, $y = 3$ and $z = -8$

a $2x + y + z$

b $3z + xy$

c $x^2 - yz$

d $\dfrac{4y}{x + z}$

Earn an Awesome passport stamp with these questions:

5 Evaluate $\dfrac{a(a + 2b)^2}{(b - a)^2}$ when $a = 2$, $b = -4$

6 Evaluate $\left[\dfrac{(x - y)^2}{(y - x)^2}\right]^2$ when $x = -1$, $y = -5$

Tables of values

These are used to show how one variable changes when another variable in a given **rule** is changed.

Complete the table of values using the rule: $b = a + 3$

a	0	1	2	3	4	5
b						

- Substitute each value of a into the rule to find b

a	0	1	2	3	4	5
b	3	4	5	6	7	8

$a = 0$	$a = 1$	$a = 2$	$a = 3$	$a = 4$	$a = 5$
$\therefore b = 0 + 3$	$\therefore b = 1 + 3$	$\therefore b = 2 + 3$	$\therefore b = 3 + 3$	$\therefore b = 4 + 3$	$\therefore b = 5 + 3$
$= 3$	$= 4$	$= 5$	$= 6$	$= 7$	$= 8$

Complete the table of values using the rule: $y = \frac{x}{3}$

x	3	6	9	12	15	18
y	1	2	3	4	5	6

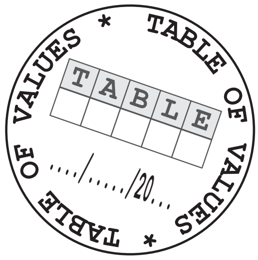
$x = 3$	$x = 6$	$x = 9$	$x = 12$	$x = 15$	$x = 18$
$\therefore y = 3 \div 3$	$\therefore y = 6 \div 3$	$\therefore y = 9 \div 3$	$\therefore y = 12 \div 3$	$\therefore y = 15 \div 3$	$\therefore y = 18 \div 3$
$= 1$	$y = 2$	$y = 3$	$y = 4$	$y = 5$	$y = 6$

Complete the table of values using the rule: $m = 3n - 1$

n	0	1	2	3	4	5
m	-1	2	5	8	11	14

$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
$\therefore m = 3 \times 0 - 1$	$\therefore m = 3 \times 1 - 1$	$\therefore m = 3 \times 2 - 1$	$\therefore m = 3 \times 3 - 1$	$\therefore m = 3 \times 4 - 1$	$\therefore m = 3 \times 5 - 1$
$= -1$	$= 2$	$= 5$	$= 8$	$= 11$	$= 14$

Table of values



1 Complete each table of values using the given rule.

a $u = v + 2$

v	0	1	2	3	4
u					

b $c = 2d$

d	0	1	2	3	4
c					

c $g = 4h - 3$

h	1	2	3	4	5
g					

d $y = \frac{x}{2} + 1$

x	2	4	6	8	10
y					

2 Draw lines to match each table of values with the correct matching rule.

a	0	2	4	6	8
b	2	3	4	5	6

a	1	2	3	4	5
b	1	6	11	16	21

a	0	1	2	3	4
b	0	3	6	9	12

a	0	1	2	3	4
b	3	5	7	9	11

- $b = 2a + 3$
- $b = \frac{a + 4}{2}$
- $b = 3a$
- $b = 5a - 4$

3 Have a go at figuring out the rule used for each table of values below and fill in the gaps.

a Rule:

x	0	1	2	3	4
y	5	6			9

b Rule:

m	0	1	2	3	4
n	0	4		12	

c Rule:

p	0	1	2	3	4
q	-3		-1		1

d Rule:

c		1		5	6
d	-5	-1	3		19

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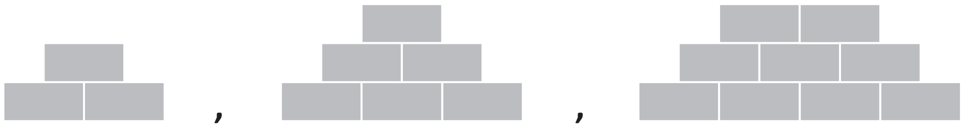
Number patterns

There are a lot of patterns in the world and it is a useful skill to be able to work them out.

" , ..." at the end means the diagrams continue to change following the same pattern.

Look at the following patterns of bricks laid by a builder over a three minute period:



1^{st} minute , 2^{nd} minute , 3^{rd} minute ,...

(i) Describe the number pattern of bricks laid by the builder every minute:

Pattern: The builder lay's three bricks in the first minute and then another 3 every minute thereafter

(ii) Write a number pattern for the total number of bricks laid after every minute:

Number Pattern: $3 \quad , \quad 6 \quad , \quad 9 \quad , \ldots$

The number pattern formed can be displayed using a table of values:

Look at the increasing arrow sign pattern below:

1^{st} sign , 2^{nd} sign , 3^{rd} sign ,...

(i) Describe the number pattern formed by the arrow signs:

Pattern: The first sign has three arrows, then each following sign increases by two arrows

(ii) Complete a table of values for the first three arrow signs:

Sign number	1	2	3
Number of arrows	3	5	7

Number pattern for the arrows used in each sign increases by two arrows each time.

(iii) How many arrows would be in the 6^{th} sign of the pattern?

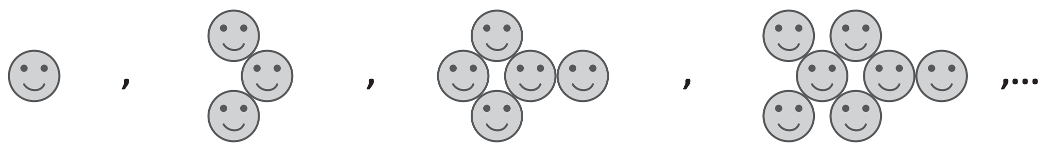
First six values in the number pattern are: $3 \underbrace{\quad , \quad}_{2} 5 \underbrace{\quad , \quad}_{2} 7 \underbrace{\quad , \quad}_{2} 9 \underbrace{\quad , \quad}_{2} 11 \underbrace{\quad , \quad}_{2} 13$

$\therefore$ there would be 13 arrows in the 6^{th} sign

Number patterns

1. For each of these pattern diagrams:
   (i) Describe the number pattern formed by the shapes
   (ii) Write a number pattern for the total number of shapes used to make the first five diagrams

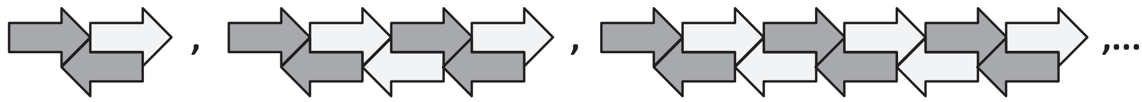
a



(i)

(ii) ________ , ________ , ________ , ________ , ________ ,...

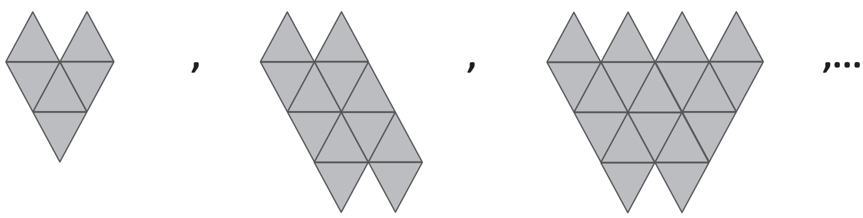
b

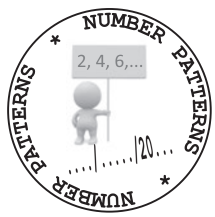


(i)

(ii) ________ , ________ , ________ , ________ , ________ ,...

c





(i)

(ii) ________ , ________ , ________ , ________ , ________ ,...

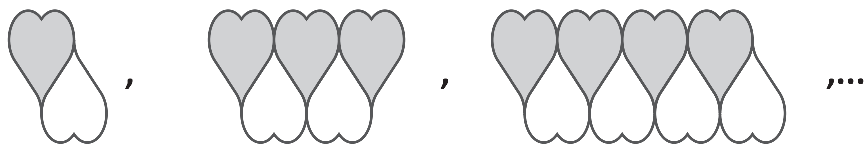
Number patterns

2 For each of these pattern diagrams:

(i) Complete a table of values for the first 4 diagrams

(ii) Write down how many shapes are needed for the 7th diagram

a



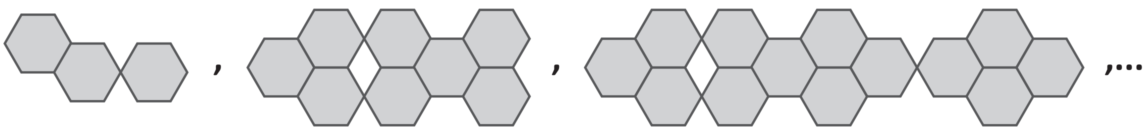
1st 2nd 3rd

(i)

Diagram number	1	2	3	4
Number of hearts				

(ii) Number of hearts needed for the 7th diagram = ___________

b



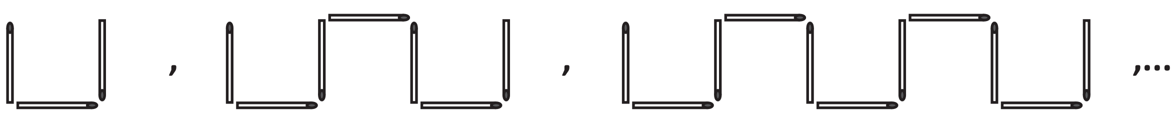
1st 2nd 3rd

(i)

Diagram number	1	2	3	4
Number of hexagons				

(ii) Number of hexagons needed for the 7th diagram = ___________

c



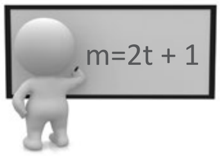
1st 2nd 3rd

(i)

Diagram number	1	2	3	4
Number of matchsticks				

(ii) Number of matchsticks needed for the 7th diagram = ___________

Modelling number patterns



Modelling a number pattern is the fancy way Mathematicians say:
'find the algebra rule for the pattern'.

These examples use the number of shapes **and** matchsticks in each pattern to find the rule.

Find the algebraic rule for the matchstick pattern below:

Number of triangles (t):	1 ,	2 ,	3	,...
Number of matchsticks (m):	3 ,	6 ,	9	,...

Pattern: Starting with 3 matchsticks, the number of matchsticks goes up by 3 with each triangle added on

$\therefore$ The number of matchsticks in each diagram equals 3 x the number of triangles in the diagram

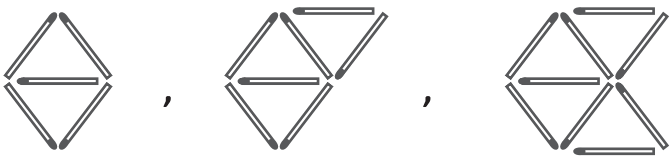
Using algebra, this is: $\boxed{m = 3t}$ —— The general rule

Completing a table of values can help to find the general rule:

Number of triangles (t)	1	2	3
Number of matchsticks (m)	3	6	9

Number of matchsticks (m) equals the number of triangles (t) multiplied by the constant increase → 3 3

Find the general rule for this pattern formed using matchsticks



Number of triangles (t):	2 ,	3 ,	4	,...
Number of matchsticks (m):	5 ,	7 ,	9	,...

Pattern: Starting with 5 matchsticks, the matchsticks increase by 2 for each extra triangle added on

Number of triangles (t)	2	3	4
Number of matchsticks (m)	5	7	9

2 2 ← Multiply t by 2: $\therefore m = 2t$

Checking with the first shape, this time we need to put +1
into the rule to get the correct number of matchsticks

$\therefore$ $\boxed{m = 2t + 1}$ —— The general rule

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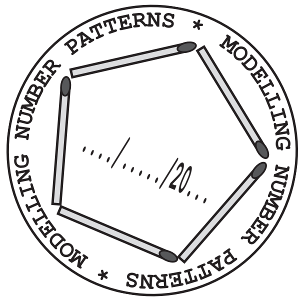
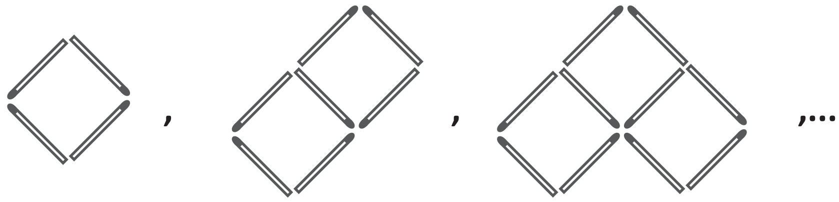
Modelling number patterns



Write down the general rule for each of the following matchstick number patterns:

1



Let s be the number of squares and m the number of matchsticks

Number of squares (s)			
Number of matchsticks (m)			

General rule:

$m = \square \times s + \square$

2

Let t be the number of triangles and m the number of matchsticks

Number of triangles (t)			
Number of matchsticks (m)			

General rule:

$m = \square\, t + \square$

3

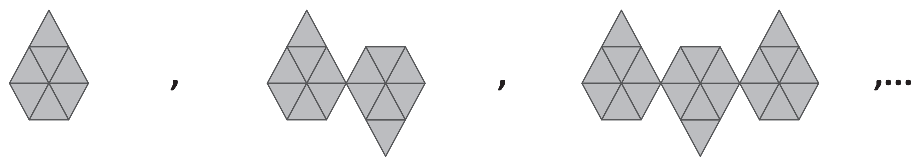
Let r be the number of grey rings and c the number of circles drawn

Number of grey rings (r)	1		
Number of circles drawn (c)	2		

General rule:

$c = \square\, r + \square$

4



Let p be the number of pentagonal shapes and t the number of triangles used

Number of pentagonal shapes (p)			
Number of triangles (t)			

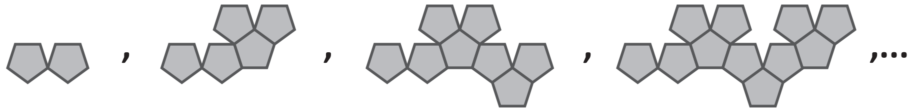
General rule:

$t = \square\, p$

More number pattern modelling

The diagram number (n^{th} diagram) and the number of shapes in each diagram is used for these questions.

Find the general rule for this pattern formed using pentagons



n^{th} diagram (n)	1^{st},	2^{nd},	3^{rd},	4^{th}	,...
Pentagons (p) used:	2,	5,	8,	11	,...

Pattern: Starting with 2 pentagons, the number of pentagons goes up by 3 for each following diagram

Diagram number (n)	1	2	3	4
Number of pentagons (p)	2	5	8	11

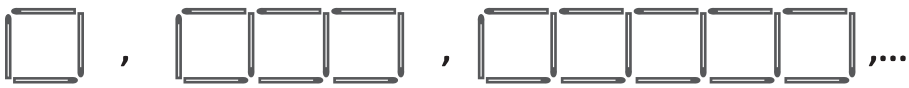
3 3 3 ← Multiply n by 3: $\therefore p = 3n$

Checking with the first shape, we need to put –1 into the rule to get the correct number of pentagons

$\therefore$ $\boxed{p = 3n - 1}$ — The general rule

This method also works for matchstick patterns.

Find the general rule for this pattern formed using matchsticks



n^{th} diagram (n)	1^{st},	2^{nd},	3^{rd}	,...
matchsticks (m) used:	4,	10,	16	,...

Pattern: Starting with 4 matchsticks, the number of matchsticks goes up by 6 for each following diagram

Diagram number (n)	1	2	3
Number of matchsticks (m)	4	10	16

6 6 ← Multiply n by 6: $\therefore m = 6n$

Checking with the first pattern, we need to put –2 into the rule to get the correct number of matchsticks

$\therefore$ $\boxed{m = 6n - 2}$ — The general rule

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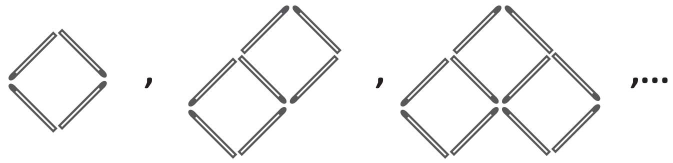
More number pattern modelling

1 Write down the general rule for each of the following matchstick number patterns:

a



1^{st} 2^{nd} 3^{rd}

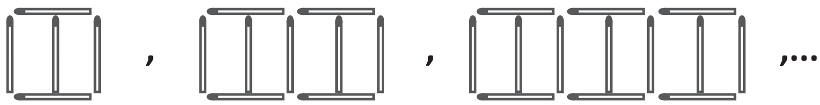
Let n be the diagram number and m the number of matchsticks

Diagram number (n)			
Number of matchsticks (m)			

General rule:

$m = \square \times n + \square$

b



1^{st} 2^{nd} 3^{rd}

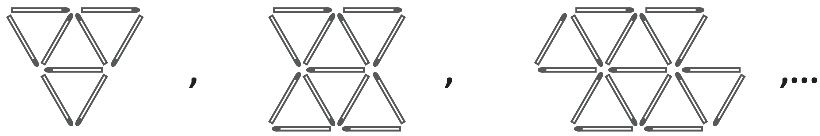
Let n be the diagram number and m the number of matchsticks

Diagram number (n)			
Number of matchsticks (m)			

General rule:

$m = \square \; n + \square$

c



1^{st} 2^{nd} 3^{rd}

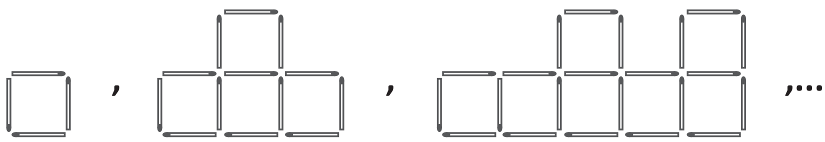
Let n be the diagram number and m the number of matchsticks

Diagram number (n)			
Number of matchsticks (m)			

General rule:

$m = \square \; n + \square$

d



1^{st} 2^{nd} 3^{rd}

Let n be the diagram number and m the number of matchsticks

Diagram number (n)			
Number of matchsticks (m)			

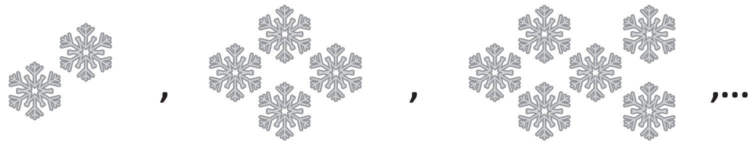
General rule:

$m = \square \; n \; \square \; \square$

More number pattern modelling

2 Write down the general rule for each of the following number patterns:

a



1^{st} 2^{nd} 3^{rd}

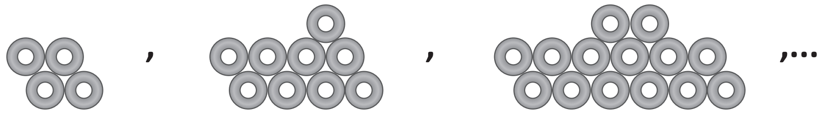
Let s be the number of snow flakes and n the n^{th} diagram

Diagram number (n)			
Number of snow flakes (s)			

General rule:

$s = \square \times n$

b



1^{st} 2^{nd} 3^{rd}

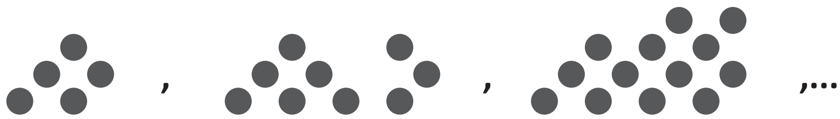
Let t be the number of tyres and n the n^{th} diagram

Diagram number (n)			
Number of tyres (t)			

General rule:

$t = \square\, n - \square$

c



1^{st} 2^{nd} 3^{rd}

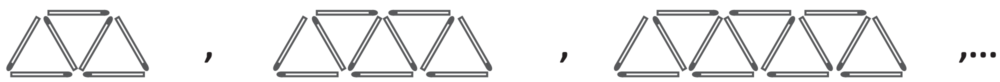
Let d be the number of dots and n the n^{th} diagram

Diagram number (n)			
Number of dots (d)			

General rule:

$d = \square \times \square \; \square \; \square$

d



1^{st} 2^{nd} 3^{rd}

Let t be the number of triangles formed and n the n^{th} diagram

Diagram number (n)			
Number of triangles (t)			

General rule:

$\square = \square \times \square \; \square \; \square$

There are actually two number patterns here, the other involves the number of matchsticks used. See if you can work it out!

Using the general rule

Substitution into the general rule is used to answer questions about the n^{th} diagram in a pattern.

Find the general rule for the parallelogram pattern:

Let p be the number of parallelograms and n the n^{th} diagram

n	1	2	3
p	2	4	6

2 2

$\therefore$ $\boxed{p = 2n}$ general rule

How many parallelograms are there in the 20^{th} diagram of the pattern above?

$\therefore p = 2 \times 20$ Substitute $n = 20$ into the general rule

$= 40$ parallelograms

Find the general rule for the matchstick number pattern:

Let m be the number of matchsticks and n the n^{th} diagram

n	1	2	3
m	3	8	13

5 5

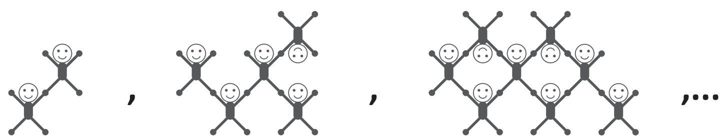
$\therefore$ $\boxed{m = 5n - 2}$ general rule

How many matchsticks are there in the 8^{th} diagram?

$\therefore m = 5 \times 8 - 2$ Substitute $n = 8$ into the general rule

$= 38$ matchsticks

Find the general rule for the amazing stick gymnast pattern:



Let g be the number of gymnasts and n the n^{th} diagram

n	1	2	3
g	2	5	8

3 3

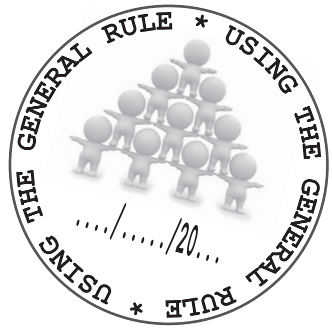
$\therefore$ $\boxed{g = 3n - 1}$ general rule

How many gymnasts are there in the 30^{th} pattern?

$\therefore g = 3 \times 30 - 1$ Substitute $n = 30$ into the general rule

$= 89$ gymnasts

Using the general rule



1 **a** Every time Niamh kicked a goal (g) the team score (s) increased by 2.
The general rule for this is given by:

$$s = 2g$$

How many points did Niamh score after kicking $g = 8$ goals?

b If the total number of chickens (c) that crossed the road after each minute (m) is given by the general rule:

$$c = 5m - 3$$

How many chickens had crossed the road when $m = 7$ minutes?

c The total number of shirts (s) tried on by customers (c) in a store is represented by the general rule:

$$s = 2c + 1$$

How many shirts had been tried on when there were $c = 12$ customers?

d The total number of vegetarian meals (v) ordered (on average) in a restaurant by diners (d) is given by the general rule:

$$v = \frac{d}{3}$$

How many vegetarian meals were ordered on a night with $d = 36$ diners?

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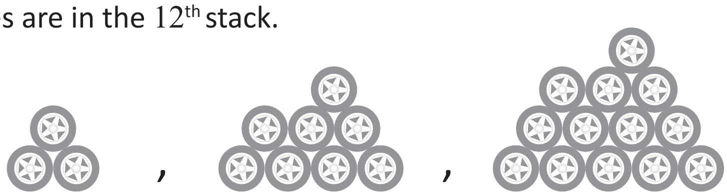
Using the general rule

2 The stacked tyres below form a number pattern. Find the general rule and then calculate how many tyres are in the 12^{th} stack.



1^{st} stack , 2^{nd} stack , 3^{rd} stack , …

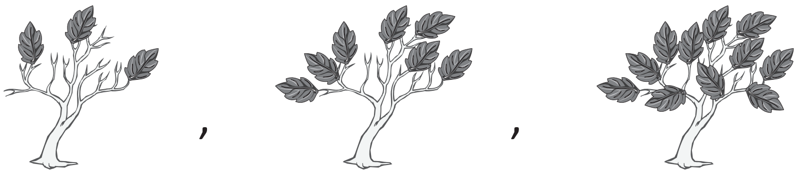
Let t be the number of tyres and n the n^{th} stack of tyres.

n			
t			

General rule:

Tyres in the 12^{th} stack:

3 New leaves are appearing on a tree each day forming a number pattern.
Find the general rule and calculate how many leaves there are on the 10^{th} day.



Day 1 , Day 2 , Day 3 , …

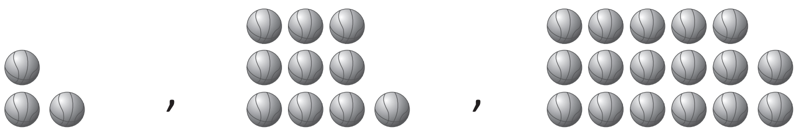
Let l be the number of leaves and n the n^{th} day.

n			
l			

General rule:

Leaves on the 10^{th} day:

4 The basketballs represent the number of good shots during each training session.
The good shots are increasing by the same amount each time.
How many good shots are made during the 8^{th} session?



Session 1 , Session 2 , Session 3 , …

Let s be the number of good shots and n the n^{th} training session.

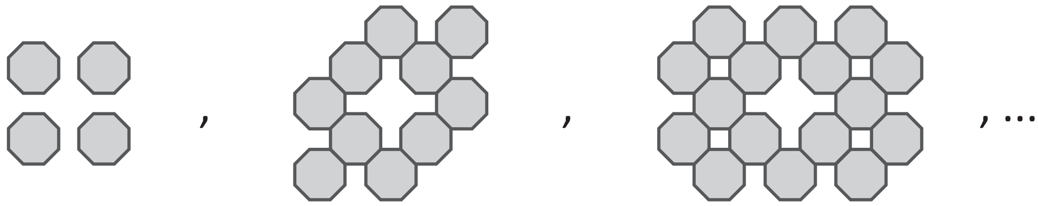
n			
s			

General rule:

Good shots in the 8^{th} session:

Using the general rule

5 A tiler is laying out some octagonal tiles in the following number pattern:



1st diagram, 2nd diagram, 3rd diagram, ...

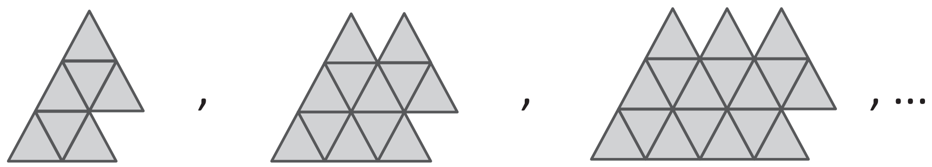
How many tiles will be laid in the 12^{th} diagram?
let t be the number of tiles laid and n the n^{th} diagram.

n			
t			

General rule:

Tiles laid in the 12^{th} diagram:

6 Triangles have been stacked to form an increasing number pattern:



7 triangles, 12 triangles, 17 triangles, ...

Find the general rule and calculate the number of triangles needed for the 15^{th} shape.
Let t be the number of trangles and n the n^{th} shape.

n			
t			

General rule:

Triangles in the 15^{th} shape:

Here is a summary of the important things to remember for algebra basics

Words and symbols

Algebra uses letters or symbols called **variables**. Each part in an algebraic expression is called a **term**.

$$x + 6$$

Variable term (can change) — Constant term (can't change)

$x + 6 = 10$ equal sign '=' means $x + 6$ has the same value as 10

The equal sign makes this an **equation**. The value of x must be 4 to be correct

---

Multiplication

Multiplications can be:

Simplified ⟶ $6 \times n = 6n$ $x \times y = xy$ **Expanded** ⟶ $6n = 6 \times n$ $xy = x \times y$

Division

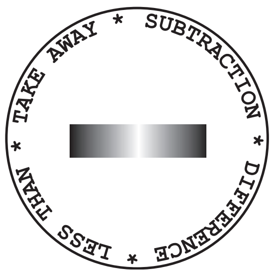
It sometimes helps to write divisions as a fraction first when simplifying: $a \div b = \frac{a}{b}$

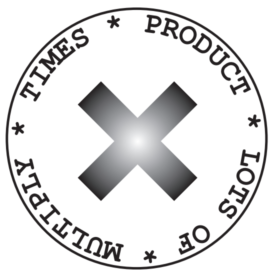
Brackets can be hidden when writing a division as a fraction: $y \div (4 + x) = \frac{y}{4 + x}$

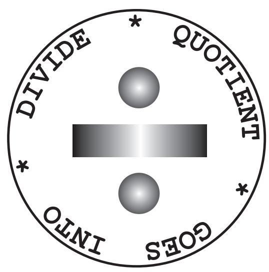
When doing the **reverse**, brackets must be **put in**: $\frac{3 - m}{6} = (3 - m) \div 6$

Phrases as algebraic expressions

To solve problems with algebra we use **variables** to turn the problem into an algebraic rule (or relationship).







Addition and subtraction

Only **Like** terms can be added or subtracted.

Like terms: x and $-x$ — Like terms; $3b$ and b — Like terms; $2y$ and $-5y$ — Like terms

Not Like terms: a and b — Not Like terms; p and p^2 — Not Like terms; $2x$ and $-5y$ — Not Like terms

Table of values

These show how one variable changes when another variable in a given rule is changed

Rule: $y = 2x + 2$

x	1	2	3
y	4	6	8

$x = 1$
$\therefore y = 2 \times 1 + 2$
$= 4$

$x = 2$
$\therefore y = 2 \times 2 + 2$
$= 6$

$x = 3$
$\therefore y = 2 \times 3 + 2$
$= 8$

Modelling number patterns

This is a fancy way Mathematicians say "find the algebra rule for the pattern"

The rule can be found using two methods:

1. Comparing the diagram number with the number of shapes in it.

2. Comparing the number of shapes with the number of objects used to make each diagram.

Tables of values help with both methods

n	1	2	3
s	4	6	8

+2 +2 ← This means 'n' is multiplied by 2 in the rule

After looking at the first values of $n = 1$ and $s = 4$, the rule must be: $s = 2n + 2$

Using the general rule

The number of shapes/objects in a particular part of the pattern is found by substituting into the general rule.

How many Squares (s) are there in the 20^{th} pattern if $s = 2n + 2$?

when $n = 20$, $s = 2 \times 20 + 2 = 42$ squares.

Words and symbols

1. a The variable is b b The variable is m c The variable is k d The variable is a

2. $3 + x =$ $x \div x = 1$ $12 \times g =$ $3 \times x + 6 =$

3.

$14 - k$ if $x = 4$	20
$4 \times m$ if $m = 5$	10
$27 \div a$ if $a = 3$	9
$1 + 37 \times z$ if $z = 3$	7

4. a $c = 8$ b $h = 12$ c $k = 18$ d $y = 6$

Multiplication

1. a $14k$ b u c $5pr$ d $12jkl$ e $18b^2$ f $12jkl$

2. a $4 \times p \times q$ b $4 \times a \times a$ c $3 \times m \times m \times n$

3. a 14 b 3 c 30 d 36

Division

1. a $\frac{2}{d}$ b $\frac{a}{c}$ c $\frac{5}{r+3}$ d $\frac{y+z}{z}$

2. a $w \div 4$ b $c \div (3 + a)$ c $6 \div (3x + 2)$ d $(x - y) \div (v + w)$

3. a $a \div 3$ b $b \div 2c$ c $3x \div 4y$ d $(m + n) \div 3p$

Mixed simplifying concepts

1. a $\frac{5a}{4}$ b $\frac{m}{4+n}$ c $\frac{mn}{abc}$ d $\frac{16p}{9q}$ e $\frac{x^2}{y+2x}$ f $\frac{d^2f}{11+ef}$

Mixed simplifying concepts

2. a $2 \times d \div 3$ b $(a + 4) \div b$ c $(q - r) \div (9 \times q)$ d $l \times l \div (j - k)$ e $5 \times b \times b \div (a \times a + 2 \times b)$ f $7 \times x \times x \times y \times z \div (x + 7 \times y)$

Phrases as algebraic expressions

1. a $n + 7$ b $9 - n$ c $6 \times n + 1 = 6n + 1$ d $4 \times n = 4n$ e $\frac{n+2}{3}$ f $n^2 - 6$ g $2(n - 5)$ h $2n - 8$ i $10 + \frac{n}{2}$ j $n(n + 5)$

2. a Correct b Incorrect c Correct d Incorrect e Incorrect f Correct g Incorrect h Correct i Incorrect j Correct

Addition and subtraction

1. a $10a$ b $8u$ c $5r$ d $-3g$ e $-2m$ f $-9x$ g $13y$ h $5p$

2. a $25m + 9n$ b $14a + 11b$ c $16x + 24y$ d $6d - 5c$ e $18e + 2a$ f $-2g - 4h$

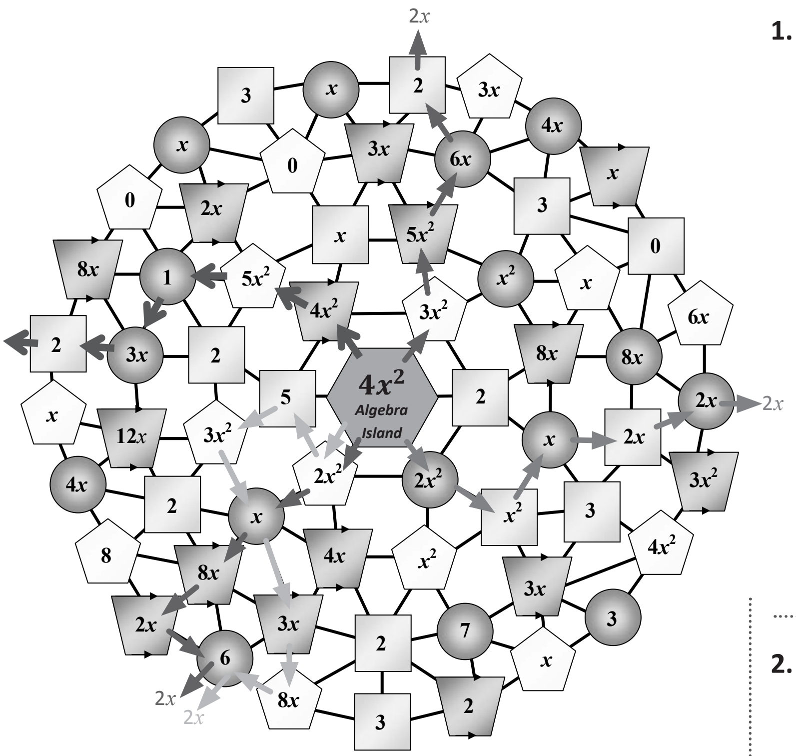
Grouping like terms

1. a $10a + 7b$ b $11p^2 + 22p$ c $-23m$ d $4y - 13x$ e $12p + 8q$ f $16a^2 + 4b - 3a$

2. a $\frac{11y}{y+2x}$ b $\frac{-p^2-5p}{12}$

3. a $\frac{4x-3y}{7x-2y}$ b $\frac{8a+6b}{5a^2}$

Escape from Algebra Island Puzzle



Bringing all the previous concepts together

1. a 18 b 3
   c 9 d 2

2. a 8 b 27
   c 8 d 5

3. a 54 b 30
   c 4 d −9

4. a 7 b −6
   c 60 d −6

5. 2

6. 1

Tables of Values

1. a

v	0	1	2	3	4
u	2	3	4	5	6

b

d	0	1	2	3	4
c	0	2	4	6	8

c

h	1	2	3	4	5
g	1	5	9	13	17

d

x	2	4	6	8	10
y	2	3	4	5	6

2.

a	0	2	4	6	8
b	2	3	4	5	6

a	1	2	3	4	5
b	1	6	11	16	21

a	0	1	2	3	4
b	0	3	6	9	12

a	0	1	2	3	4
b	3	5	7	9	11

$b = 2a + 3$

$b = \dfrac{a+4}{2}$

$b = 3a$

$b = 5a - 4$

3. a

x	0	1	2	3	4
y	5	6	7	8	9

b

m	0	1	2	3	4
n	0	4	8	12	16

c

p	0	1	2	3	4
q	-3	-2	-1	0	1

d

c	0	1	2	5	6
d	-5	-1	3	15	19

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Number patterns

1. a (i) Starting with one smiley face in the first diagram, 2 smiley faces are added to each diagram every time.

   (ii) 1 , 3 , 5 , 7 , 9 , ...

   b (i) Starting with three arrows in the first diagram, 4 arrows are added to each diagram every time

   (ii) 3 , 7 , 11 , 15 , 19 , ...

   c (i) Starting with 6 triangles to form the first diagram, 6 triangles are added to each diagram every time.

   (ii) 6 , 12 , 18 , 24 , 30 , ...

2. a (i)

Diagram number	1	2	3	4
Number of hearts	2	5	8	11

   (ii) 20

   b (i)

Diagram number	1	2	3	4
Number of hexagons	3	8	13	18

   (ii) 33

   c (i)

Diagram number	1	2	3	4
Number of matchsticks	3	7	11	15

   (ii) 27

Modelling Number Patterns

1. a $m = \boxed{3} \times s + \boxed{1}$

   b $m = \boxed{2}\, t + \boxed{1}$

   c $c = \boxed{1}\, r + \boxed{1}$

   d $t = \boxed{7}\, p$

More number pattern modelling

1. a $m = \boxed{3} \times n + \boxed{1}$

   b $m = \boxed{4}\, n + \boxed{1}$

   c $m = \boxed{4}\, n + \boxed{5}$

   d $m = \boxed{9}\, n \boxed{-} \boxed{5}$

2. a $s = \boxed{2} \times n$

   b $t = \boxed{5}\, n - \boxed{1}$

   c $d = \boxed{4} \times \boxed{n} \boxed{+} \boxed{1}$

   d $\boxed{t} = \boxed{2} \times \boxed{n} \boxed{+} \boxed{1}$

Using the general rule

1. a 16 points b 32 chickens
   c 25 shirts d 12 vegetarion meals

2. 58

3. 39

4. 52

5. 70

6. 77

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