Algebra Basics

Solutions

Curriculum Ready

ACMNA: 133, 175, 176, 177, 179

Mathletics

www.mathletics.com

First edition printed 2009 in Australia.

A catalogue record for this book is available from 3P Learning Ltd.

ISBN 978-1-921861-25-3

Published 3P Learning Ltd

For more copies of this book, contact us at: www.3plearning.com/contact/

Designed 3P Learning Ltd

Page 3 questions

Words and symbols

1 a The variable is b

b The variable is m

c The variable is k

d The variable is a

2

$5 \times w = 30$ $\quad$ $x \div x = 1$ $\quad$ $32 - 2 \times d = 16$

$3 + x =$ $\quad$ $12 \times g =$

$200 \div s = 25$ $\quad$ $3 \times x + 6 =$

3

$11 - x$ if $x = 4$	20
$4 \times m$ if $m = 5$	10
$27 \div a$ if $a = 3$	9
$1 + 3 \times z$ if $z = 3$	7

4 a $c = 8$ $\quad$ b $h = 12$ $\quad$ c $k = 18$ $\quad$ d $y = 6$

Page 5 questions

Multiplication

1 a $2 \times 7 \times k = 14 \times k$
$= 14k$

b $u \times 1 = u$

c $5 \times r \times p = 5pr$

d $n \times m \times m = m^2 n$

e $6 \times b \times 3 \times b = 6 \times 3 \times b \times b$
$= 18 \times b^2$
$= 18b^2$

f $4 \times j \times l \times 3 \times k = 4 \times 3 \times j \times l \times k$
$= 12 \times j \times k \times l$
$= 12jkl$

Page 5 questions

2 **a** $4pq = 4 \times p \times q$ **b** $4a^2 = 4 \times a \times a$ **c** $3m^2n = 3 \times m \times m \times n$

3 Combo Time!

a $$\begin{aligned} 3x + 2 &= 3 \times 4 + 2 \\ &= 12 + 2 \\ &= 14 \end{aligned}$$

b $$\begin{aligned} 15 - 2b &= 15 - 2 \times 6 \\ &= 15 - 12 \\ &= 3 \end{aligned}$$

c $$\begin{aligned} 3 \times 5g &= 3 \times 5 \times 2 \\ &= 30 \end{aligned}$$

d $$\begin{aligned} 4m^2 &= 4 \times 3^2 \\ &= 4 \times 9 \\ &= 36 \end{aligned}$$

Page 7 questions

Division

1 **a** $2 \div d = \dfrac{2}{d}$

b $a \div c = \dfrac{a}{c}$

c $5 \div (r + 3) = \dfrac{5}{r + 3}$

d $(y + z) \div z = \dfrac{y + z}{z}$

2 **a** $\dfrac{w}{4} = w \div 4$

b $\dfrac{c}{3 + a} = c \div (3 + a)$

c $\dfrac{6}{3x + 2} = 6 \div (3x + 2)$

d $\dfrac{x - y}{v + w} = (x - y) \div (v + w)$

3 **a** $$\begin{aligned} \frac{2a}{6} &= \frac{a}{3} \\ &= a \div 3 \end{aligned}$$

b $$\begin{aligned} \frac{6b}{12c} &= \frac{b}{2c} \\ &= b \div 2c \end{aligned}$$

c $$\begin{aligned} \frac{15x}{20y} &= \frac{3x}{4y} \\ &= 3x \div 4y \end{aligned}$$

d $$\begin{aligned} \frac{4(m + n)}{12p} &= \frac{m + n}{3p} \\ &= (m + n) \div 3p \end{aligned}$$

Page 8 questions

Mixed simplifying concepts

1

a $5 \times a \div 4 = 5a \div 4$
$= \dfrac{5a}{4}$

b $1 \times m \div (4 + n) = m \div (4 + n)$
$= \dfrac{m}{4 + n}$

c $n \times m \div (b \times a \times c) = mn \div abc$
$= \dfrac{mn}{abc}$

d $(8 \times 2p) \div (3 \times 3q) = 16p \div 9q$
$= \dfrac{16p}{9q}$

e $x \times x \div (y + 2x) = x^2 \div (y + 2x)$
$= \dfrac{x^2}{y + 2x}$

f $d \times f \times d \div (11 + f \times e) = d^2 f \div (11 + ef)$
$= \dfrac{d^2 f}{11 + ef}$

2

a $\dfrac{2d}{3} = 2d \div 3$
$= 2 \times d \div 3$

b $\dfrac{a + 4}{b} = (a + 4) \div b$

c $\dfrac{q - r}{9q} = (q - r) \div 9q$
$= (q - r) \div (9 \times q)$

d $\dfrac{l^2}{j - k} = l^2 \div (j - k)$
$= l \times l \div (j - k)$

e $\dfrac{5b^2}{a^2 + 2b} = 5b^2 \div (a^2 + 2b)$
$= 5 \times b \times b \div (a \times a + 2 \times b)$

f $\dfrac{7xyz}{x + 7y} = 7xyz \div (x + 7y)$
$= 7 \times x \times y \times z \div (x + 7 \times y)$

Page 10 questions

Phrases as algebraic expressions

a $n + 7$

b $9 - n$

c $6 \times n + 1 = 6n + 1$

d $4 \times n = 4n$

Page 10 questions

1

e $(n + 2) \div 3 = \frac{n+2}{3}$

f $n^2 - 6$

g $(n - 5) \times 2 = 2(n - 5)$

h $2 \times n - 8 = 2n - 8$

i $10 + n \div 2 = 10 + \frac{n}{2}$

j $n \times (n + 5) = n(n + 5)$

Page 11 questions

2

a	Correct	Incorrect	
b	Correct	Incorrect	expression should be: $n - 4$
c	Correct	Incorrect	
d	Correct	Incorrect	expression should be: $(4 + n) \div 9$ or $\frac{4+n}{9}$
e	Correct	Incorrect	expression should be: $(n \div 5) + n$ or $\frac{n}{5} + n$
f	Correct	Incorrect	
g	Correct	Incorrect	expression should be: $n + (3 - \frac{n}{2})$
h	Correct	Incorrect	
i	Correct	Incorrect	expression should be: $3 \times n^2 = 3n^2$
j	Correct	Incorrect	

Page 13 questions

Addition and subtraction

1 a $a + 9a = 1a + 9a$
$= 10a$

b $3u + 5u = 8u$

c $14r - 9r = 5r$

d $4g - 7g = -3g$

e $6m - 8m = -2m$

f $-11x + 2x = -9x$

g $7y + 2y + 4y = 13y$

h $30p - 15p - 10p = 5p$

2 a $\boxed{13m} + 9n \boxed{+12m} = 13m + 12m + 9n$
$= 25m + 9n$
Like terms

b $14a \boxed{+ b} \boxed{+ 10b} = 14a + 11b$
Like terms

c $16x \boxed{+ 9y} \boxed{+ 15y} = 16x + 24y$
Like terms

d $\boxed{9d} - 5c \boxed{- 3d} = 9d - 3d - 5c$
$= 6d - 5c$
Like terms

e $\boxed{7e} \boxed{+ 11e} + 2a = 18e + 2a$
Like terms

f $\boxed{13g} \boxed{- 15g} - 4h = -2g - 4h$
Like terms

Page 15 questions

Grouping like terms

Like terms

1 a $9a \; \boxed{+\,3b} \; +\,a \; \boxed{+\,4b} = 9a + a \; \boxed{+\,3b + 4b}$

Like terms

$= 10a + 7b$

Like terms

b $4p^2 \; \boxed{+\,3p} \; \boxed{+\,19p} \; +\,7p^2 = 4p^2 + 7p^2 \; \boxed{+\,3p + 19p}$

Like terms

$= 11p^2 + 22p$

Like terms

c $n \; \boxed{-\,11m} \; -\,n \; \boxed{-\,12m} = n - n \; \boxed{-\,11m - 12m}$

Like terms

$= -\,23m$

Like terms

d $3y \; \boxed{-\,5x} \; +\,y \; \boxed{-\,8x} = 3y + y \; \boxed{-\,5x - 8x}$

Like terms

$= 4y - 13x$

Like terms

e $9p \; \boxed{-\,4q} \; +\,3p \; \boxed{+\,12q} = 9p + 3p \; \boxed{-\,4q + 12q}$

Like terms

$= 12p + 8q$

e $14a^2 + 4b - 3a + 2a^2 = 14a^2 + 2a^2 + 4b - 3a$

Like terms

$= 16a^2 + 4b - 3a$

Combo time!

2 a $11y \div (2y + 2x - y) = 11y \div (y + 2x)$

Like terms

$= \dfrac{11y}{y + 2x}$

b $(7p^2 - 5p - 8p^2) \div 12 = (-\,p^2 - 5p) \div 12$

Like terms

$= \dfrac{-\,p^2 - 5p}{12}$

Page 15 questions

Like terms

3 **a** $(2x - 3y + 2x) \div (\boxed{4x}\boxed{+3x} - 2y) = (4x - 3y) \div (7x - 2y)$

Like terms

$$= \frac{4x - 3y}{7x - 2y}$$

b $(2 \times 4a + 3 \times 2b) \div (3 \times a \times a + 2a^2) = (8a + 6b) \div (3a^2 + 2a^2)$

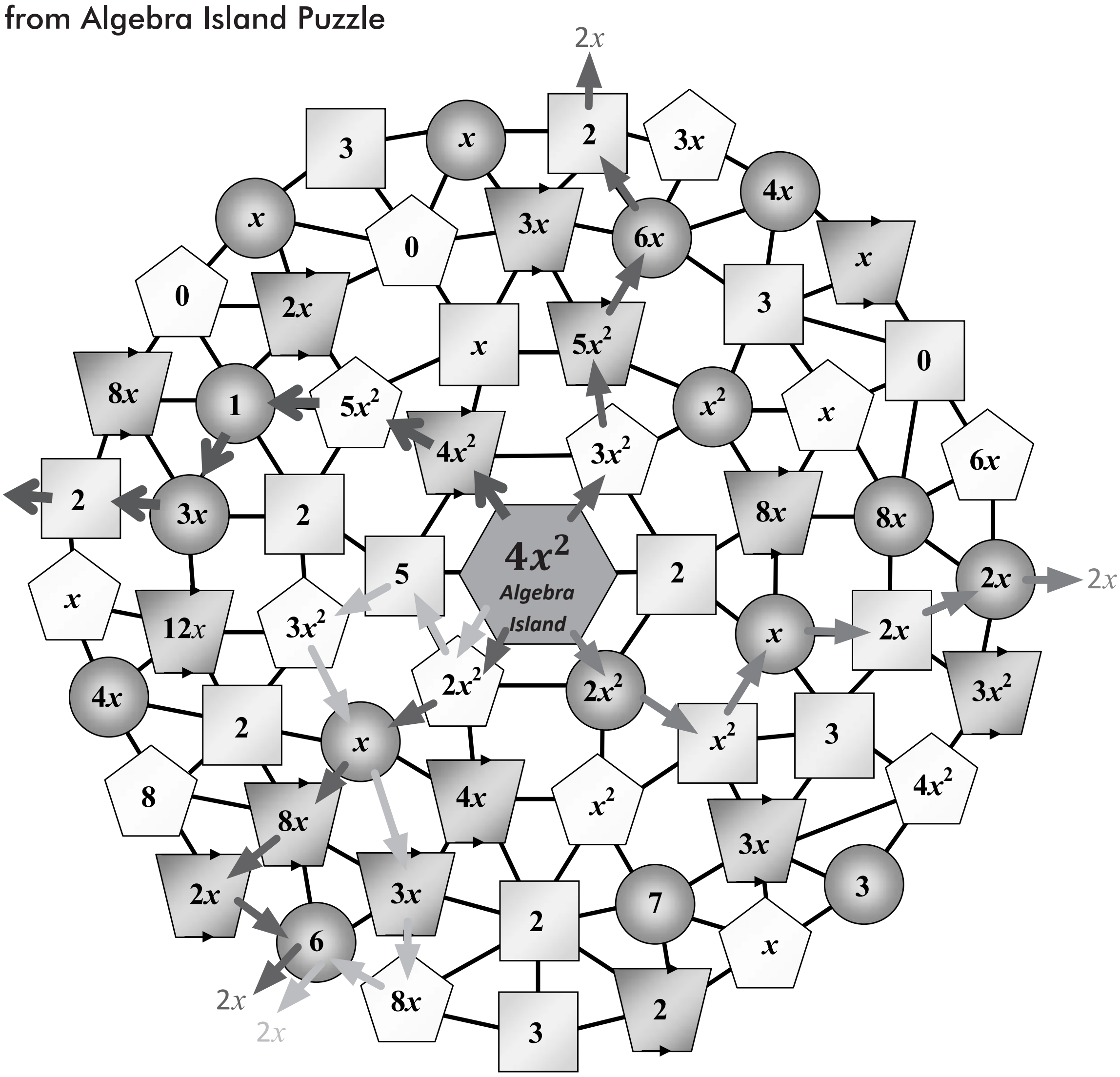
Like terms

$$= (8a + 6b) \div 5a^2$$

$$= \frac{8a + 6b}{5a^2}$$

Page 16 questions

Escape from Algebra Island Puzzle



Page 19 questions

Bringing all the previous concepts together

1

a $4v + 2 = 4 \times 4 + 2$
$= 18$

b $24 \div 2v = 24 \div (2 \times 4)$
$= 24 \div 8$
$= 3$

c $10 - \frac{v}{4} = 10 - \frac{4}{4}$
$= 10 - 1$
$= 9$

d $\frac{2v + 6}{7} = (2v + 6) \div 7$
$= (2 \times 4 + 6) \div 7$
$= 14 \div 7$
$= 2$

2

a $a + 2b = -2 + 2 \times 5$
$= -2 + 10$
$= 8$

b $3b - 6a = 3 \times 5 - 6 \times (-2)$
$= 15 - (-12)$
$= 27$

c $\frac{24}{a + b} = 24 \div (a + b)$
$= 24 \div (-2 + 5)$
$= 24 \div 3$
$= 8$

d $\frac{a^2 b}{4} = a^2 b \div 4$
$= ((-2)^2 \times 5) \div 4$
$= (4 \times 5) \div 4$
$= 20 \div 4$
$= 5$

3

a Like terms

$c \boxed{+ d} + 2c \boxed{+ 3d} = 3c + 4d$

Like terms

$= 3 \times 6 + 4 \times 9$
$= 18 + 36$
$= 54$

b Like terms

$2c \boxed{+ d} + 3c \boxed{- d} = 5c$

Like terms

$= 5 \times 6$
$= 30$

c $\frac{2d - c}{d - c} = (2d - c) \div (d - c)$
$= (2 \times 9 - 6) \div (9 - 6)$
$= 12 \div 3$
$= 4$

d $(c - d) \times (2c - d) = (c - d) \times (2c - d)$
$= (6 - 9) \times (2 \times 6 - 9)$
$= (-3) \times 3$
$= -9$

Page 20 questions

4 a $$\begin{aligned}2x + y + z &= 2 \times 6 + 3 + (-8)\\ &= 12 + 3 - 8\\ &= 7\end{aligned}$$

b $$\begin{aligned}3z - xy &= 3 \times (-8) + 6 \times 3\\ &= -24 + 18\\ &= -6\end{aligned}$$

c $$\begin{aligned}x^2 - yz &= 6^2 - (3 \times (-8))\\ &= 36 - (-24)\\ &= 60\end{aligned}$$

d $$\begin{aligned}\frac{4y}{x+z} &= 4y \div (x+z)\\ &= 4 \times 3 \div (6 + (-8))\\ &= 12 \times (-2)\\ &= -6\end{aligned}$$

5 $\dfrac{a(a+2b)^2}{(b-a)^2} = a(a+2b)^2 \div (b-a)^2$ when $a = 2,\ b = -4$

$$\begin{aligned}&= 2 \times (2 + 2 \times (-4))^2 \div (-4-2)^2\\ &= 2 \times (-6)^2 \div (-6)^2\\ &= 2 \times 36 \div 36\\ &= 2\end{aligned}$$

6 $\left(\dfrac{(x-y)^2}{(y-x)^2}\right)^2 = ((x-y)^2 \div (y-x)^2)^2$ when $x = -1,\ y = -5$

$$\begin{aligned}&= ((-1-(-5))^2 \div (-5-(-1))^2)^2\\ &= ((-1+5)^2 \div (-5+1)^2)^2\\ &= (4^2 \div (-4)^2)^2\\ &= (16 \div 16)^2\\ &= (1)^2\\ &= 1\end{aligned}$$

Page 22 questions

Tables of Values

1 **a** $u = v + 2$

v	0	1	2	3	4
u	2	3	4	5	6

b $c = 2d$

d	0	1	2	3	4
c	0	2	4	6	8

c $g = 4h - 3$

h	1	2	3	4	5
g	1	5	9	13	17

d $y = \frac{x}{2} + 1$

x	2	4	6	8	10
y	2	3	4	5	6

2

a	0	2	4	6	8
b	2	3	4	5	6

a	1	2	3	4	5
b	1	6	11	16	21

a	0	1	2	3	4
b	0	3	6	9	12

a	0	1	2	3	4
b	3	5	7	9	11

$b = 2a + 3$

$b = \frac{a + 4}{2}$

$b = 3a$

$b = 5a - 4$

3 **a** Rule: $y = x + 5$

x	0	1	2	3	4
y	5	6	7	8	9

b Rule: $n = 4 \times m$ or $n = 4m$

m	0	1	2	3	4
n	0	4	8	12	16

c Rule: $q = p - 3$

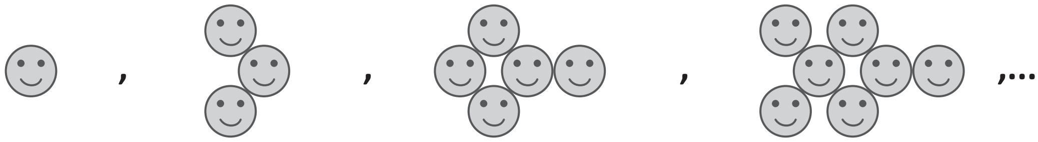
p	0	1	2	3	4
q	-3	-2	-1	0	1

d Rule: $d = 4c - 5$

c	0	1	2	5	6
d	-5	-1	3	15	19

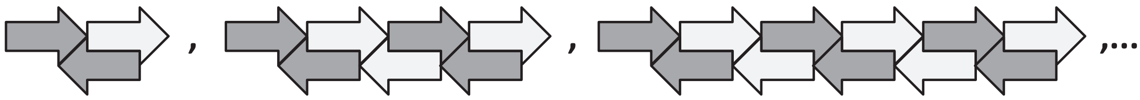
Page 24 questions

Number patterns

1 **a** 

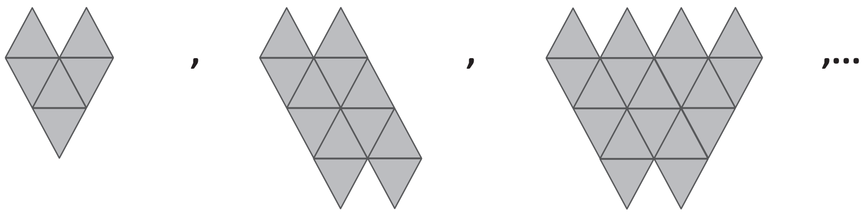
(i) Starting with one smiley face in the first diagram, 2 smiley faces are added to each diagram every time

(ii) Number pattern for first five smiley diagrams: 1 , 3 , 5 , 7 , 9 , ...

b 

(i) Starting with three arrows in the first diagram, 4 arrows are added to each diagram every time

(ii) Number pattern for first five arrow diagrams: 3 , 7 , 11 , 15 , 19 , ...

c 

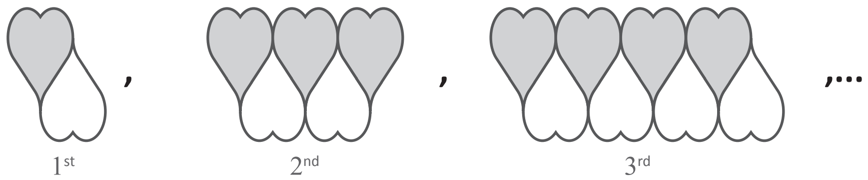
(i) Starting with 6 triangles to form the first diagram, 6 triangles are added to each diagram every time.

(ii) Number pattern for first five arrow diagrams: 6 , 12 , 18 , 24 , 30 , ...

Page 25 questions

Number patterns

2 a

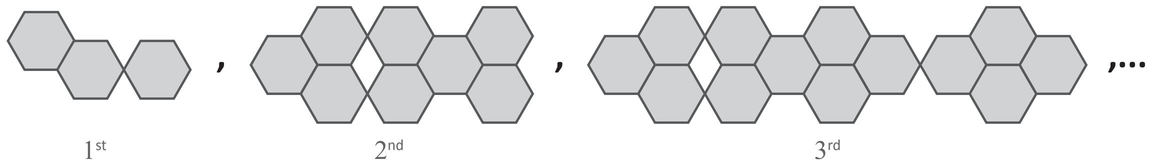


(i)

Diagram number	1	2	3	4
Number of hearts	2	5	8	11

(ii) Number of hearts needed for the 7^{th} diagram = 20

b

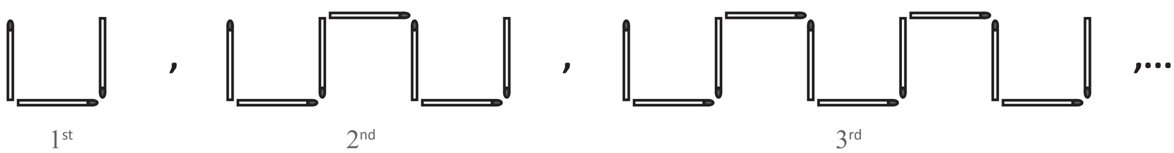


(i)

Diagram number	1	2	3	4
Number of hexagons	3	8	13	18

(ii) Number of hexagons needed for the 7^{th} diagram = 33

c



(i)

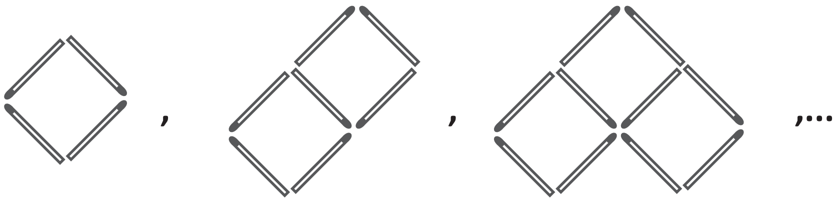
Diagram number	1	2	3	4
Number of matchsticks	3	7	11	15

(ii) Number of matchsticks needed for the 7^{th} diagram = 27

Page 27 questions

Modelling Number Patterns

1 **a**



Number of squares (s)	1	2	3
Number of matchsticks (m)	4	7	10

General rule:

$m = \boxed{3} \times s + \boxed{1}$

b

Number of triangles (t)	3	4	5
Number of matchsticks (m)	7	9	11

General rule:

$m = \boxed{2}\, t + \boxed{1}$

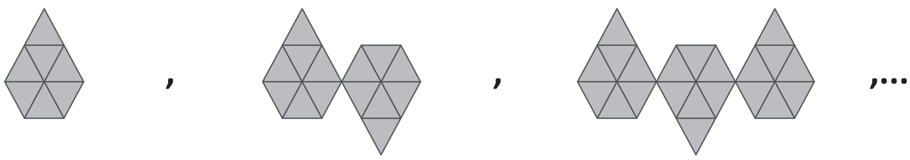
c

Number of grey rings (r)	1	2	3
Number of circle drawn (c)	2	3	4

General rule:

$c = \boxed{1}\, r + \boxed{1}$

d



Number of pentagonal shapes (p)	1	2	3
Number of triangles (t)	7	14	21

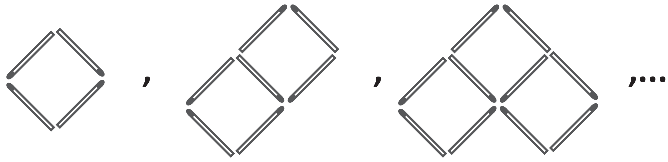
General rule:

$t = \boxed{7}\, p$

Page 29 questions

More number pattern modelling

1 a



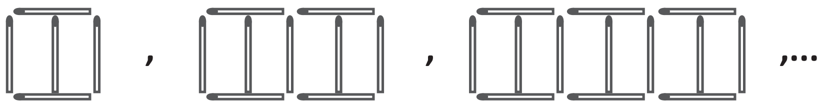
1st 2nd 3rd

Diagram number (n)	1	2	3
Number of matchsticks (m)	4	7	10

General rule:

$m = \boxed{3} \times n + \boxed{1}$

b



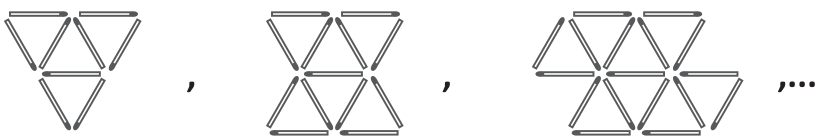
1st 2nd 3rd

Diagram number (n)	1	2	3
Number of matchsticks (m)	5	9	13

General rule:

$m = \boxed{4}\ n + \boxed{1}$

c



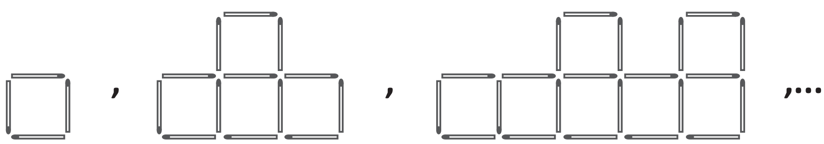
1st 2nd 3rd

Diagram number (n)	1	2	3
Number of matchsticks (m)	9	13	17

General rule:

$m = \boxed{4}\ n + \boxed{5}$

d



1st 2nd 3rd

Diagram number (n)	1	2	3
Number of matchsticks (m)	4	13	22

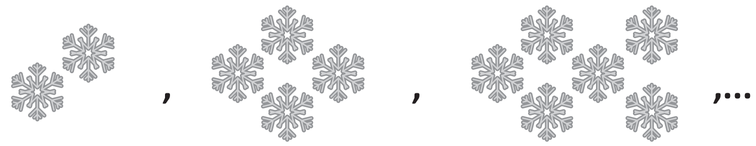
General rule:

$m = \boxed{9}\ n \boxed{-} \boxed{5}$

Page 30 questions

More number pattern modelling

2 **a**



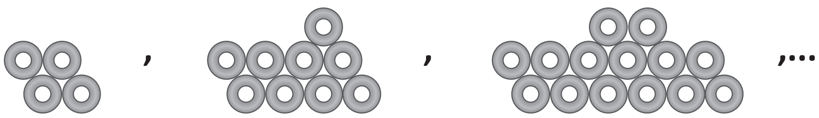
1^{st} 2^{nd} 3^{rd}

Diagram number (n)	1	2	3
Number of snow flakes (s)	2	4	6

General rule:

$s = \boxed{2} \times n$

b



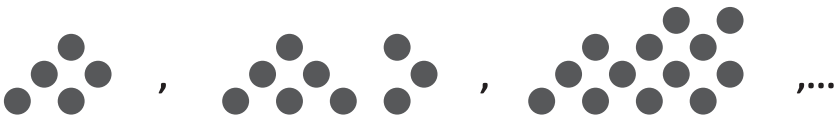
1^{st} 2^{nd} 3^{rd}

Diagram number (n)	1	2	3
Number of tyres (t)	4	9	14

General rule:

$t = \boxed{5}\, n - \boxed{1}$

c



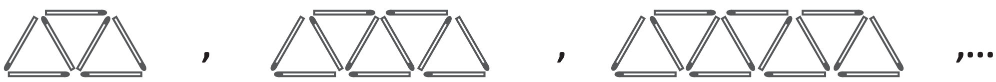
1^{st} 2^{nd} 3^{rd}

Diagram number (n)	1	2	3
Number of dots (d)	5	9	13

General rule:

$d = \boxed{4} \times \boxed{n} \boxed{+} \boxed{1}$

d



1^{st} 2^{nd} 3^{rd}

Diagram number (n)	1	2	3
Number of triangles (t)	3	5	7

General rule:

$\boxed{t} = \boxed{2} \times \boxed{n} \boxed{+} \boxed{1}$

Page 32 questions

Using the general rule

1 **a** $s = 2g$

Niamh scored: $s = 2 \times 8$

$= 16$ points

b $c = 5m - 3$

The number of chickens that had crossed the road: $c = 5 \times 7 - 3$

$= 32$ chickens

c $s = 2c + 1$

The number of shirts tried on: $s = 2 \times 12 + 1$

$= 25$ shirts

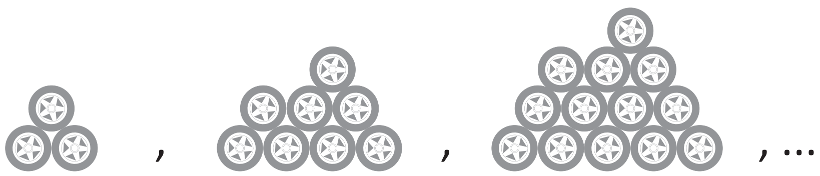
d $v = \frac{d}{3}$

The number of vegetarian meals ordered: $v = \frac{36}{3}$

$= 12$ vegetarian meals

Page 33 questions

2



1^{st} stack, 2^{nd} stack, 3^{rd} stack, …

n	1	2	3
t	3	8	13

General rule:

$t = 5n - 2$

Tyres in the 12^{th} stack:

$t = 5 \times 12 - 2$

$= 58$

3

Day 1, Day 2, Day 3, …

n	1	2	3
l	3	7	11

General rule:

$l = 4n - 1$

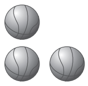
Leaves on the 10^{th} day:

$l = 4 \times 10 - 1$

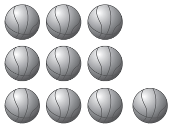
$= 39$

Page 33 questions

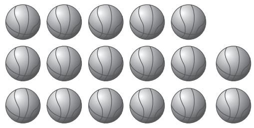
4



Session 1



Session 2



Session 3

n	1	2	3
s	3	10	17

General rule:

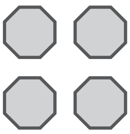
$s = 7n - 4$

Good shots in the 8th session:

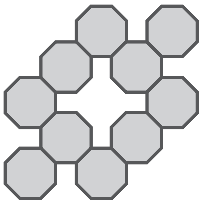
$s = 7 \times 8 - 4$
$= 52$

Page 34 questions

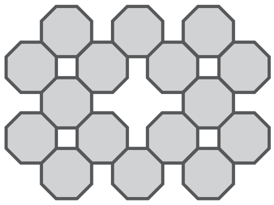
5



1st diagram



2nd diagram



3rd diagram

n	1	2	3
t	4	10	16

General rule:

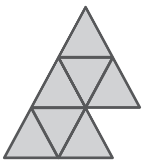
$t = 6n - 2$

Tiles laid in the 12th diagram:

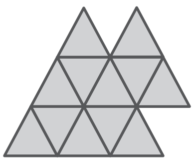
$t = 6 \times 12 - 2$
$= 70$

Remember me?

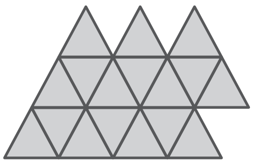
6



7 triangles



12 triangles



17 triangles

n	1	2	3
t	7	12	17

General rule:

$t = 5n + 2$

Triangles in the 15th shape:

$t = 5 \times 15 + 2$
$= 77$