Percentage Calculations

Curriculum Ready

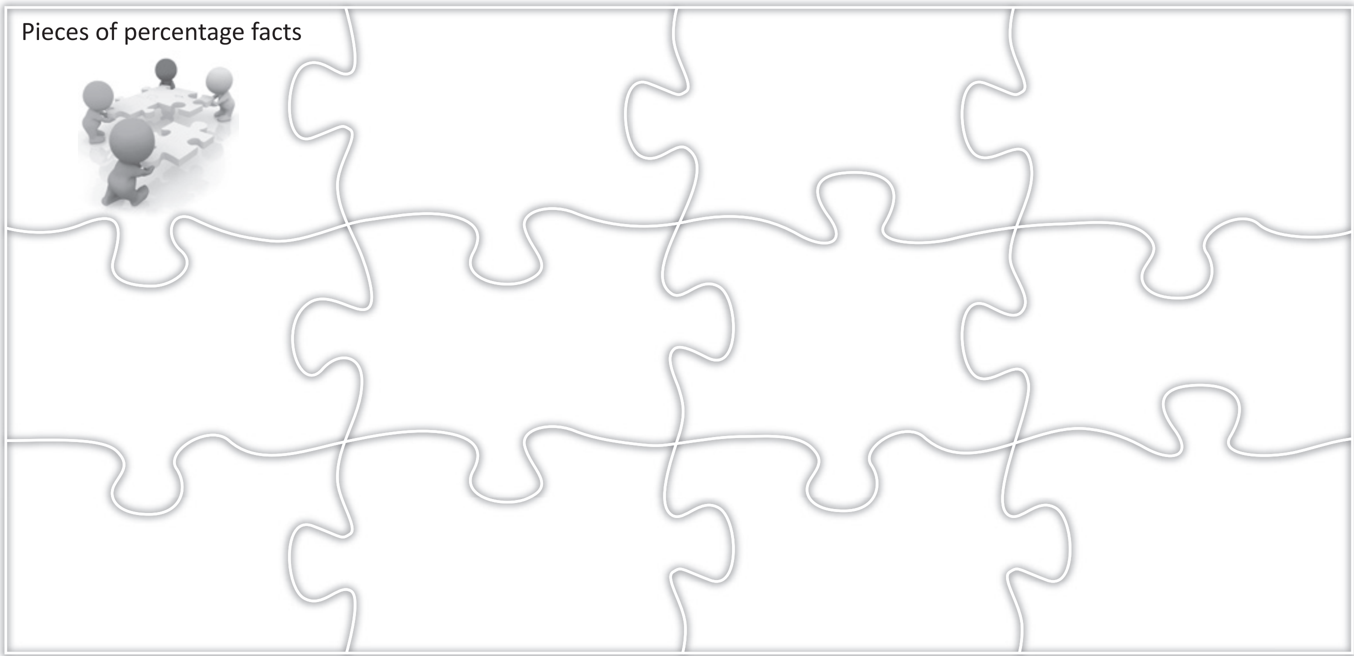
ACMNA: 157, 158, 187, 189

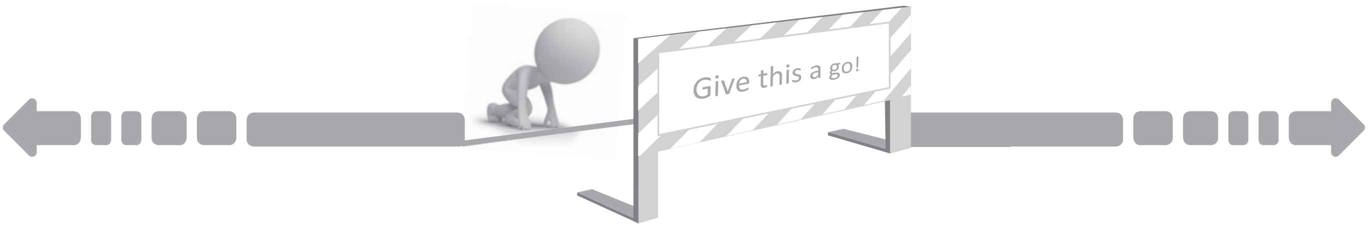
Mathletics

www.mathletics.com

This booklet is all about percentage calculations and where they are used in our daily lives.

What do you already know about percentages? Fill in the pieces with some facts you already have.





Q Look at this nutritional information for a packet of food.

Nutritional Information

Serving size: 5 g

	Avg Qty per Serve	Avg Qty per 100g
Energy	78 kJ	1550 kJ
Protein	0.4 g	8.0 g
Fat, total	0.2 g	4.4 g
-saturated	0.0 g	0.4 g
Carbohydrate	3.8 g	75.0 g
-sugars	1.3 g	25.9 g
Sodium	8 mg	165 mg
Vitamin C	75 mg	1500 mg
Vitamin A	1539 IU	28390 IU

The standard method used by nutritionists to estimate our minimum daily grams of protein needed is to multiply the body weight in kilograms by 0.8.

a Calculate your minimum daily requirement of protein.

b How many servings of this food will a 75 kg adult need to eat to get their minimum daily requirement of protein?

Work through the book for a great way to do this

Fractions and percentages

Percent (%) = for every (per) 100 (cent)

$\therefore$ 50% means 50 for every 100, which as a fraction = $\frac{50}{100} = \frac{1}{2}$ ← Always write in simplest form

To change fractions to a percentage, write as an **equivalent fraction** with a denominator of 100.

- Proper fractions (for amounts smaller than the whole and percentages $< 100\%$)

$\frac{\times 25}{\times 25}$

$\frac{1}{4}$ ← **Initial fraction** **Equivalent fraction** → $\frac{25}{100} = 25\%$

(100 in denominator)

$\frac{\div 25}{\div 25}$

- Mixed numerals and improper fractions (for amounts larger than the whole and percentages $> 100\%$)

$\frac{\times 50}{\times 50}$

$1\frac{1}{2} = \frac{3}{2}$ ← **Initial fraction** **Equivalent fraction** → $\frac{150}{100} = 150\%$

Mixed Numeral Improper fractions (100 in denominator)

$\frac{\div 50}{\div 50}$

Another simple way to change fractions to a percentage is by multiplying the fraction by 100.

$$\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = 25\% \qquad \frac{3}{2} = \left(\frac{3}{2} \times 100\right)\% = 150\%$$

Convert these fractions and percentages using the method given

(i) 75% (Equivalent fraction method)

$75\% = \frac{75}{100}$ Percent signs means 'over 100'

$= \frac{3}{4}$ Simplify fraction

(ii) $\frac{8}{200}$ (Equivalent fraction method)

$\frac{8}{200} = \frac{8 \div 2}{200 \div 2}$ Divide numerator and denominator by 2

$= \frac{4}{100}$ Equivalent fraction with 100 as denominator

$= 4\%$

(iii) $\frac{6}{5}$ (Multiplication method)

$\frac{6}{5} = \left(\frac{6}{5} \times 100\right)\%$ Multiply fraction by 100

$= 120\%$

Fractions and percentages

1 Write these fractions as a percentage.

a $\frac{3}{100} = \square\ \%$ b $\frac{41}{100} = \square\ \%$ c $\frac{110}{100} = \square\ \%$ d $\frac{200}{100} = \square\ \%$

2 Write these percentages as a fraction.

a $7\% = \frac{\square}{\square}$ b $89\% = \frac{\square}{\square}$ c $117\% = \frac{\square}{\square}$ d $336\% = \frac{\square}{\square}$

3 Write these percentages as a fraction with 100 in the denominator and then simplify.

a $20\% = \frac{\square}{100} = \frac{\square}{\square}$ Simplified

b $15\% = \frac{\square}{100} = \frac{\square}{\square}$ Simplified

c $80\% = \frac{\square}{100} = \frac{\square}{\square}$ Simplified

d $24\% = \frac{\square}{\square} = \frac{\square}{\square}$ Simplified

e $42\% = \frac{\square}{\square} = \frac{\square}{\square}$ Simplified

f $96\% = \frac{\square}{\square} = \frac{\square}{\square}$ Simplified

g $125\% = \frac{\square}{\square} = \frac{\square}{\square}$ Simplified improper

h $180\% = \frac{\square}{\square} = \frac{\square}{\square}$ Simplified improper

i $350\% = \frac{\square}{\square} = \frac{\square}{\square}$ Simplified improper

4 Write these as equivalent fractions with a denominator of 100 and then as a percentage.

a $\frac{24}{300} = \frac{24 \div \square}{300 \div \square} = \frac{\square}{100} = \square\ \%$

b $\frac{48}{200} = \frac{48 \div \square}{200 \div \square} = \frac{\square}{\square} = \square\ \%$

c $\frac{175}{500} = \frac{175 \div \square}{500 \div \square} = \frac{\square}{\square} = \square\ \%$

Fractions and percentages

5 Write these as equivalent fractions with a denominator of 100 and then as a percentage.

a $\frac{3}{10} = \frac{3 \times \square}{10 \times \square} = \frac{\square}{100} = \square\,\%$

b $\frac{12}{25} = \frac{12 \times \square}{25 \times \square} = \frac{\square}{\square} = \square\,\%$

c $\frac{6}{5} = \frac{6 \times \square}{5 \times \square} = \frac{\square}{\square} = \square\,\%$

6 Write these as equivalent mixed numerals with a denominator of 100 and then as a percentage.

a $3\frac{1}{2} = \frac{\boxed{7} \times \boxed{50}}{2 \times \boxed{50}} = \frac{\boxed{350}}{\boxed{100}} = \boxed{350}\,\%$

b $2\frac{1}{4} = \frac{\square \times \square}{4 \times \square} = \frac{\square}{\boxed{100}} = \square\,\%$

c $1\frac{2}{5} = \frac{\square \times \square}{5 \times \square} = \frac{\square}{\boxed{100}} = \square\,\%$

7 Use the multiplication method to write these fractions as percentages.

a $\frac{1}{2} = \square\,\%$

b $\frac{1}{5} = \square\,\%$

c $\frac{8}{25} = \square\,\%$

d $\frac{13}{50} = \square\,\%$

e $\frac{5}{2} = \square\,\%$

f $\frac{15}{20} = \square\,\%$

8 Change these to improper fractions and use the multiplication method to change to a percentage.

a $2\frac{2}{5} = \frac{\square}{\square} = \square\,\%$

b $3\frac{3}{4} = \frac{\square}{\square} = \square\,\%$

c $1\frac{7}{20} = \frac{\square}{\square} = \square\,\%$

d $4\frac{9}{25} = \frac{\square}{\square} = \square\,\%$

Fractions and percentages

Sometimes the equivalent fraction with a denominator of 100 has a decimal numerator.

$\frac{7}{200}$ ← **Initial fraction** $\underset{\frac{\times 2}{\times 2}}{\overset{\frac{\div 2}{\div 2}}{\rightleftarrows}}$ **Equivalent fraction** (100 in denominator) → $\frac{3.5}{100}$

So the equivalent percentages in all forms (decimal, mixed number and improper fraction) are:

$\therefore \frac{7}{200} = 3.5\% = 3\frac{1}{2}\% = \frac{7}{2}\%$

Decimal | Mixed numeral | Improper fraction

9 Write these as a percentage in decimal and mixed numeral form.

a $\frac{13}{500} = \frac{\square}{100} = \square\% = \square\frac{\square}{\square}\%$ (Decimal, Mixed numeral)

b $\frac{30}{800} = \frac{\square}{100} = \square\% = \square\frac{\square}{\square}\%$ (Decimal, Mixed numeral)

c $\frac{17}{400} = \frac{\square}{\square} = \square\% = \square\frac{\square}{\square}\%$ (Decimal, Mixed numeral)

d $\frac{26}{500} = \frac{\square}{\square} = \square\% = \square\frac{\square}{\square}\%$ (Decimal, Mixed numeral)

e $\frac{47}{500} = \frac{\square}{\square} = \square\% = \square\frac{\square}{\square}\%$ (Decimal, Mixed numeral)

f $\frac{42}{400} = \frac{\square}{\square} = \square\% = \square\frac{\square}{\square}\%$ (Decimal, Mixed numeral)

10 Write these as a percentage in decimal and improper fraction form.

a $\frac{3}{200} = \frac{\square}{100} = \square\% = \frac{\square}{\square}\%$ (Decimal, Improper fraction)

b $\frac{7}{500} = \frac{\square}{\square} = \square\% = \frac{\square}{\square}\%$ (Decimal, Improper fraction)

c $\frac{9}{800} = \frac{\square}{\square} = \square\% = \frac{\square}{\square}\%$ (Decimal, Improper fraction)

d $\frac{9}{750} = \frac{\square}{\square} = \square\% = \frac{\square}{\square}\%$ (Decimal, Improper fraction)

Decimals and percentages

Changing between decimals and percentages is all about multiplying and dividing by 100.

Move decimal point 2 spaces right

$0.32.$

$\times 100$

0.32 ← **Decimal** **Percentage** → 32%

$\div 100$

$.32.0$

Move decimal point 2 spaces left

Let's look at two more examples moving in opposite directions.

Write these as their equivalent percentage or decimal

(i) 1%

$1\% = 1 \div 100$	Divide by 100 to get equivalent decimal
$= .01.$	Move decimal point 2 spaces left
$= 0.01$	Equivalent decimal

(ii) 0.7

$0.7 = (0.7 \times 100)\%$	Multiply by 100 to get equivalent percentage
$= 0.70.\%$	Move decimal point 2 spaces right
$= 70\%$	Equivalent percentage

(iii) 0.025

$0.025 = (0.025 \times 100)\%$	Multiply by 100 to get equivalent percentage
$= 0.02.5\%$	Move decimal point 2 spaces right
$= 2.5\%$ or $2\frac{1}{2}\%$ or $\frac{3}{2}\%$	Equivalent percentage in all forms

(iv) 101.5%

$101.5\% = 101.5 \div 100$	Divide by 100 to get equivalent decimal
$= 1.01.5\%$	Move decimal point 2 spaces left
$= 1.015$	Equivalent decimal
$= 1\frac{15}{1000} \quad \left(= 1\frac{3}{200}\right)$	Equivalent mixed numeral

Decimals and percentages

1 Write these percentages as a decimal.

a $15\% =$ ☐ b $20\% =$ ☐ c $4\% =$ ☐ d $9\% =$ ☐

e $125\% = 1.25$ f $250\% =$ ☐ g $110\% =$ ☐ h $305\% =$ ☐

2 Write these decimals as percentages.

a $0.03 =$ ☐ % b $0.16 =$ ☐ % c $1.12 =$ ☐ % d $2.45 =$ ☐ %

e $0.125 =$ ☐ % f $0.253 =$ ☐ % g $0.018 =$ ☐ % h $0.2225 =$ ☐ %

3 Write these decimals as decimal percentages and either mixed numeral or improper fractions.

a $0.015 =$ ☐ % (Decimal) $= \frac{☐}{☐}\%$ (Improper fraction)

b $0.185 =$ ☐ % (Decimal) $=$ ☐ $\frac{☐}{☐}\%$ (Mixed numeral)

c $0.012 =$ ☐ % (Decimal) $= \frac{☐}{☐}\%$ (Improper fraction)

d $0.458 =$ ☐ % (Decimal) $=$ ☐ $\frac{☐}{☐}\%$ (Mixed numeral)

4 Write these percentages as decimals, mixed numerals and improper fractions.

a $155\% =$ ☐ (Decimal) $=$ ☐ $\frac{☐}{☐}$ (Mixed numeral) $= \frac{☐}{☐}$ (Improper fraction)

b $218\% =$ ☐ (Decimal) $=$ ☐ $\frac{☐}{☐}$ (Mixed numeral) $= \frac{☐}{☐}$ (Improper fraction)

c $100.5\% =$ ☐ (Decimal) $=$ ☐ $\frac{☐}{☐}$ (Mixed numeral) $= \frac{☐}{☐}$ (Improper fraction)

d $220.4\% =$ ☐ (Decimal) $=$ ☐ $\frac{☐}{☐}$ (Mixed numeral) $= \frac{☐}{☐}$ (Improper fraction)

e $375.20\% =$ ☐ (Decimal) $=$ ☐ $\frac{☐}{☐}$ (Mixed numeral) $= \frac{☐}{☐}$ (Improper fraction)

f $125.8\% =$ ☐ (Decimal) $=$ ☐ $\frac{☐}{☐}$ (Mixed numeral) $= \frac{☐}{☐}$ (Improper fraction)

Decimals and percentages

5 Write each of these percentages in all their equivalent forms:

a 25%

b 5.5%

c $4\frac{3}{4}\%$

d $2\frac{1}{8}\%$

e 112.5%

f 237.2%

g 17.25%

h $7\frac{3}{16}\%$

i 2.375%

j $100\frac{5}{8}\%$

Recurring decimals and percentages

Recurring decimals are treated just like terminating decimals when changing to percentages.

Move decimal point 2 spaces right

$0.66.666\ldots$

$$0.\dot{6} = 0.666\ldots \longleftarrow \textbf{Decimal} \underset{\div 100}{\overset{\times 100}{\rightleftarrows}} \textbf{Percentage} \longrightarrow 66.666\ldots\% = 66.\dot{6}\%$$

$0.66.666\ldots$

Move decimal point 2 spaces left

It is usually nicer to write recurring decimals as a mixed numeral percentage where possible.

$$\therefore 66.\dot{6}\% = 66\tfrac{6}{9}\% = 66\tfrac{2}{3}\%$$

Always simplify

These tables show that there are patterns for writing simple recurring decimals as fractions:

$0.\dot{1} = \frac{1}{9}$	$0.\dot{2} = \frac{2}{9}$	$0.\dot{3} = \frac{1}{3}$	$0.\dot{4} = \frac{4}{9}$	$0.\dot{5} = \frac{5}{9}$	$0.\dot{6} = \frac{2}{3}$	$0.\dot{7} = \frac{7}{9}$	$0.\dot{8} = \frac{8}{9}$	$0.\dot{9} = \frac{9}{9} = 1$

$0.0\dot{1} = \frac{1}{90}$	$0.0\dot{2} = \frac{2}{90} = \frac{1}{45}$	and so on...

$0.2\dot{1} = \frac{2}{10} + \frac{1}{90} = \frac{19}{90}$	$0.03\dot{5} = \frac{3}{100} + \frac{5}{900} = \frac{8}{225}$	and so on...

Here are some more examples with slight differences involving recurring decimals

(i) Convert $0.00\dot{4}$ to its equivalent percentage value.

Remember: Change so the recurring pattern starts from the decimal point.

$$\begin{aligned} 0.00\dot{4} &= 0.00444\ldots \\ &= (0.00444\ldots \times 100)\% && \text{Multiply the decimal by } 100 \\ &= 0.444\ldots\% \\ &= 0.\dot{4}\% && \text{Recurring decimal percentage} \\ &= \frac{4}{9}\% && \text{Percentage form from the table} \end{aligned}$$

(ii) Convert $2\frac{2}{3}\%$ to its equivalent decimal and fraction values.

$$\begin{aligned} 2\tfrac{2}{3}\% &= 2.\dot{6}\% \\ &= 2.\dot{6} \div 100 && \text{Divide by } 100 \\ &= 0.02.666\ldots && \text{Decimal point moves two spaces to the left} \\ &= 0.02\dot{6} && \text{Equivalent recurring decimal} \end{aligned}$$

$$\begin{aligned} 0.02\dot{6} &= \frac{2}{100} + \frac{6}{900} \\ &= \frac{2}{75} && \text{Equivalent fraction} \end{aligned}$$

Recurring decimals and percentages

$0.\dot{1} = \frac{1}{9}$	$0.\dot{2} = \frac{2}{9}$	$0.\dot{3} = \frac{1}{3}$	$0.\dot{4} = \frac{4}{9}$	$0.\dot{5} = \frac{5}{9}$	$0.\dot{6} = \frac{2}{3}$	$0.\dot{7} = \frac{7}{9}$	$0.\dot{8} = \frac{8}{9}$	$0.\dot{9} = \frac{9}{9} = 1$

$0.0\dot{1} = \frac{1}{90}$	$0.0\dot{2} = \frac{2}{90} = \frac{1}{45}$	and so on...

$0.2\dot{1} = \frac{2}{10} + \frac{1}{90} = \frac{19}{90}$	$0.03\dot{5} = \frac{3}{100} + \frac{5}{900} = \frac{8}{225}$	and so on...

1 Use the tables above to help write these percentages as fractions.

a $0.\dot{1}\% = \frac{\square}{\square}\%$ b $0.\dot{5}\% = \frac{\square}{\square}\%$ c $0.0\dot{7}\% = \frac{\square}{\square}\%$ d $0.0\dot{6}\% = \frac{\square}{\square}\%$

2 Use the tables above to help write these percentages as the sum of two fractions then simplify.

a $0.1\dot{3}\% = \frac{\square}{\square} + \frac{\square}{\square}\% = \frac{\square}{\square}\%$

b $0.3\dot{8}\% = \frac{\square}{\square} + \frac{\square}{\square}\% = \frac{\square}{\square}\%$

c $0.08\dot{3} = \frac{\square}{\square} + \frac{\square}{\square}\% = \frac{\square}{\square}\%$

d $0.05\dot{7}\% = \frac{\square}{\square} + \frac{\square}{\square}\% = \frac{\square}{\square}\%$

3 Convert these mixed numeral percentages into their equivalent decimals and fractions.

a $23\frac{1}{3}\% = \square\% = \square = \frac{\square}{\square}$
Decimal form, Equivalent decimal, Fraction

b $14\frac{4}{9}\% = \square\% = \square = \frac{\square}{\square}$
Decimal form, Equivalent decimal, Fraction

c $5\frac{1}{9}\% = \square\% = \square = \frac{\square}{\square}$
Decimal form, Equivalent decimal, Fraction

d $9\frac{2}{3}\% = \square\% = \square = \frac{\square}{\square}$
Decimal form, Equivalent decimal, Fraction

Recurring decimals and percentages

4. Join each equivalent percentage, decimal and fraction with straight lines to create symmetrical triangle art.

- $0.20\dot{9}$
- $0.02\dot{1}$
- $30\frac{1}{3}\%$
- $2\frac{1}{9}\%$
- $\frac{91}{300}$
- $\frac{1}{900}$
- $0.00\dot{1}$
- $\frac{11}{300}$
- $0.20\dot{7}$
- $\frac{187}{900}$
- $20\frac{1}{9}\%$
- $\frac{91}{300}$
- $7.\dot{6}\%$
- $0.05\dot{4}$
- $0.0\dot{5}$
- $\frac{49}{900}$
- $5\frac{4}{9}\%$
- $\frac{4}{75}$
- $3\frac{2}{3}\%$
- $0.03\dot{6}$
- $\frac{23}{300}$
- $5.\dot{5}\%$
- $0.07\dot{6}$
- $\frac{5}{90}$
- $\frac{1}{9}\%$
- $5\frac{1}{3}\%$
- $0.05\dot{3}$
- $0.30\dot{3}$
- $20\frac{7}{9}\%$
- $\frac{19}{900}$

Recurring decimals and percentages

Here are some examples of the many other recurring decimals along with their equivalent fractions.

$0.\dot{0}\dot{9} = 0.0909\ldots = \frac{1}{11}$	$0.0\dot{4}\dot{5} = 0.04545\ldots = \frac{1}{22}$	$0.0\dot{3}\dot{0} = 0.03030\ldots = \frac{1}{33}$
$0.1\dot{6} = 0.1666\ldots = \frac{1}{6}$	$0.0\dot{8}\dot{3} = 0.08383\ldots = \frac{1}{12}$	$0.0\dot{5} = 0.0555\ldots = \frac{1}{18}$

Remember:
A dot is placed above the first and last digit of the recurring pattern.

To change the fraction into the recurring decimal, just divide the numerator (top) by the denominator (bottom) on your calculator.

5 Write these fractions as recurring decimals and then as equivalent percentages rounded to 1 d.p.

a $\frac{8}{11} = \square \div \square$
$= \square$ Recurring decimal
$= \square \times \square\%$
$= \square\%$
Equivalent percentage to 1 d.p.

b $\frac{19}{12} = \square \div \square$
$= \square$ Recurring decimal
$= \square \times \square\%$
$= \square\%$
Equivalent percentage to 1 d.p.

c $\frac{5}{6} = \square$ Recurring decimal
$= \square\%$ Percentage to 1 d.p.

d $\frac{2}{15} = \square$ Recurring decimal
$= \square\%$ Percentage to 1 d.p.

e $\frac{5}{18} = \square$ Recurring decimal
$= \square\%$ Percentage to 1 d.p.

f $\frac{9}{22} = \square$ Recurring decimal
$= \square\%$ Percentage to 1 d.p.

g $\frac{7}{12} = \square$ Recurring decimal
$= \square\%$ Percentage to 1 d.p.

h $\frac{13}{33} = \square$ Recurring decimal
$= \square\%$ Percentage to 1 d.p.

i $\frac{12}{11} = \square$ Recurring decimal
$= \square\%$ Percentage to 1 d.p.

j $\frac{17}{15} = \square$ Recurring decimal
$= \square\%$ Percentage to 1 d.p.

k $\frac{16}{12} = \square$ Recurring decimal
$= \square\%$ Percentage to 1 d.p.

Complementary percentages

Percent (%) = for every 100
(per) (cent)

100% = 100 for every 100

In other words, 100% represents the whole amount.

Complementary percentages split up the whole amount into two parts.

So complementary percentages add to 100%

For example, 40% of the people partying below have a hat. What percentage are not wearing hats?

People without party hats = All the people partying − the people partying with hats

$= 100\% - 40\%$

$= 60\%$

Percentage without party hats	and	percentage with party hats		are complementary percentages
60%	$+$	40%	$=$	100%

Complementary fractions and decimals add to 1 (which is 100% in decimal/ fraction form).

Find the complement for each of these

(i) 20.5%

$100\% - 20.5\% = 79.5\%$ Subtract from 100%

$\therefore$ The complement of 20.5% is 79.5%

(ii) $\frac{2}{7}$

$1 - \frac{2}{7} = \frac{5}{7}$

$\therefore$ The complement of $\frac{2}{7}$ is $\frac{5}{7}$ Subtract from fraction equivalent of 100%, 1

(iii) $0.\dot{3} (= 0.333\ldots)$

$1 - 0.\dot{3} = 0.\dot{6}$ Subtract from decimal equivalent of 100%, 1

$\therefore$ The complement of $0.\dot{3}$ is $0.\dot{6}$ Corresponding decimal places add to 9

Complementary percentages

1 Write the complement of these percentages:

a) 10% Complement = ____ %

b) 25% Complement = ____ %

c) 55% Complement = ____ %

d) 37% Complement = ____ %

e) 2% Complement = ____ %

f) 99% Complement = ____ %

2 Write the complement of these decimals:

a) 0.4 Complement = ____

b) 0.8 Complement = ____

c) 0.75 Complement = ____

d) 0.65 Complement = ____

e) 0.32 Complement = ____

f) 0.07 Complement = ____

g) $0.\dot{6}$ Complement = ____

h) $0.\dot{8}$ Complement = ____

i) $0.1\dot{4}$ Complement = ____

3 Write the complement of these fractions:

a) $\frac{1}{3}$ Complement = $\frac{\square}{\square}$

b) $\frac{3}{4}$ Complement = $\frac{\square}{\square}$

c) $\frac{4}{5}$ Complement = $\frac{\square}{\square}$

d) $\frac{1}{2}$ Complement = $\frac{\square}{\square}$

e) $\frac{7}{10}$ Complement = $\frac{\square}{\square}$

f) $\frac{13}{25}$ Complement = $\frac{\square}{\square}$

4 Write the complement of these percentages:

a) 65.5% Complement = ____ %

b) 0.5% Complement = ____ %

c) 12.75% Complement = ____ %

d) 30.15% Complement = ____ %

e) 73.12% Complement = ____ %

f) 90.99% Complement = ____ %

g) $22\frac{1}{5}\%$ Complement = ____ %

h) $90\frac{5}{6}\%$ Complement = ____ %

i) $\frac{3}{20}\%$ Complement = ____ %

Percentage of an amount

Calculate 35% of 60 — 'of' means $\times$

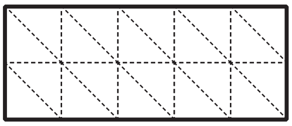
$\frac{35}{100} \times 60 = 21$ Using the equivalent fraction

or

$0.35 \times 60 = 21$ Using the equivalent decimal

These types of calculations let you to work out how much of an object the percentage represents.

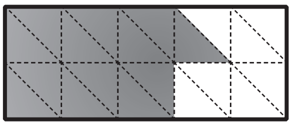
For example, paint 65% of the shape below:



This shape is divided into 20 equal triangles.

$\therefore$ The question is just asking you to calculate 65% of 20.

$\therefore 0.65 \times 20 = 13$



65% of this shape (13 triangles) has been painted.

Here are some other percentage of an amount word examples

(i) 20 people form an arrow, with the front person dressed in red and the remainder in white. If 30% of the group dressed like the person at the front, how many will be dressed in red?

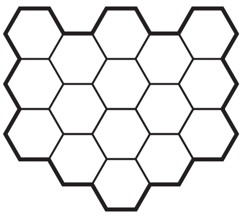
30% of $20 = \frac{30}{100} \times 20$ Multiply by equivalent fraction (or decimal)

$= \frac{6\cancel{0}\cancel{0}}{1\cancel{0}\cancel{0}}$

$= 6$ Simplified fraction

$\therefore$ 6 people in the group will be dressed in red. Answer question with a statement

(ii) If asked to shade in 0.75 of the area shown, how many hexagons will you need to shade?

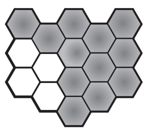


$0.75 = 75\%$, and 16 hexagons are used to make the shape.

$\therefore$ 0.75% of 16 hexagons $= 0.75 \times 16$ hexagons

$= 12$ hexagons

$\therefore$ Shading 0.75 of the area means 12 hexagons must be shaded.



Percentage of an amount

1. Complete the steps for these percentage calculations:

a. $15\% \text{ of } 40 = \dfrac{\boxed{}}{\boxed{}} \times \boxed{}$

$= \boxed{}$

b. $28\% \text{ of } 75 = \dfrac{\boxed{}}{\boxed{}} \boxed{}\ 75$

$= \boxed{}$

c. $65\% \text{ of } 60 = \left\{\boxed{} \times \boxed{}\right\} \times 60$

$= \boxed{} \times 60$

$= \boxed{}$

d. $22.5\% \text{ of } 280 = \left\{22.5 \boxed{}\ 100\right\} \boxed{} \boxed{}$

$= \boxed{} \times \boxed{}$

$= \boxed{}$

2. Calculate these percentage amounts showing all working.

a. 20% of 65

b. 60% of 35

c. 22% of 25

d. 15% of 30

e. 67.5% of 48

f. $34\frac{1}{2}\%$ of 14

These ones will result in answers larger than the original amount.

g. 125% of 12

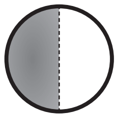
h. 220% of 40

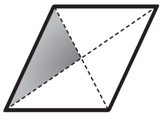
i. 150% of 15

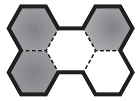
j. 175% of 79

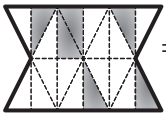
Percentage of an amount

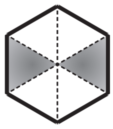
3 What percentage of these shapes has been shaded?

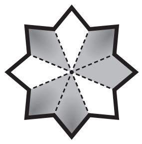
a  = %

b  = %

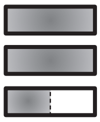
c  = %

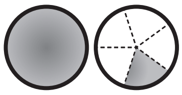
d  = %

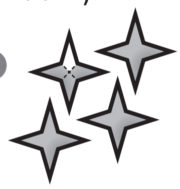
e  = %

f  = %

Hint: the next three have more than one whole shape in them (so $> 100\%$).

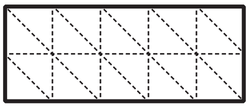
g  = %

h  = %

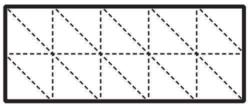
i  = %

4 Shade these shapes made using 20 identical triangles to match the given percentage values.

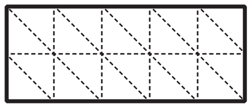
a 50%



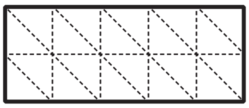
b 20%



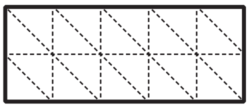
c 25%



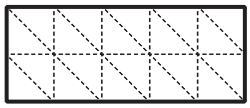
d 75%



e 30%

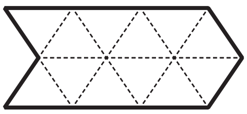


f 65%

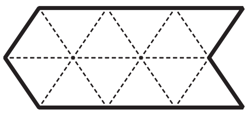


5 Shade the percentage of these shapes that match the given decimal values.

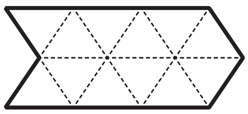
a 0.75



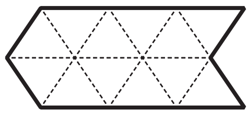
b 0.25



c $0.\dot{3}$

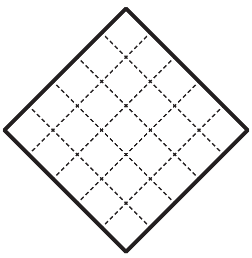


d $0.58\dot{3}$

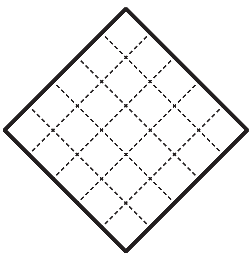


6 Shade the percentage of these shapes that match the given values.
Some shapes will need to be divided up into smaller parts to shade correctly.

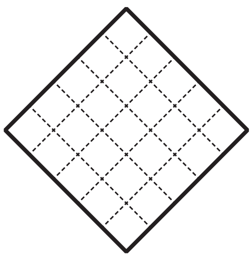
a 90%



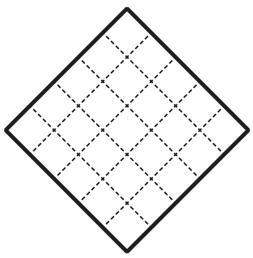
b 25%



c 35%



d 49%



Percentage of an amount

6 Show all working for these questions and answer with a statement:

a At dinner function, 40% of the 95 diners chose the vegetarian main course dish. How many diners chose the non-vegetarian main course option?

b During one 8-hour work day, Mitch spent 30% of the time in meetings with his team. How many hours did he spend in meetings on that day?

c Only 15.5% of the $12\,800$ tickets on sale for a concert were still available one hour after tickets went on sale. How many tickets are there left after 1 hour?

d A kayaker used the right-side paddle 0.42 of the time and the left side paddle 0.58 of the time while travelling down a river. How many times did the right paddle enter the water if there were a total of 3650 paddle strokes made?

7 This one will earn you an awesome passport stamp!
Use working to show that 40% of 75 the same as 75% of 40. Briefly explain why this is true for all percentage calculations.

Percentage of an amount

Complementary percentages can be used to simplify percentage problems:

Find the complement for each of these

At a call centre, 32% of the 50 people working there are currently not taking phone calls. How many people are currently taking phone calls?

The percentage of people taking phone calls is the complement of 32%.

The complement of 32% is 68%.

$\therefore$ 68% of the 50 people are taking calls.

$\therefore 0.68 \times 50 = 34$

$\therefore$ 34 people are currently taking phone calls in the call centre.

8. 75% of a 64GB (gigabytes) music device is filled with tunes from the owner's music collection. How many gigabytes does the owner have left available for more music downloads?

9. A shopping list contained 96 different items. If the shopper has only 12.5% of the listed items remaining to collect, how many items from the shopping list are in the trolley?

10. A 900 piece jigsaw puzzle has 0.43 of the pieces placed correctly into the puzzle. Use the complementary decimal to calculate how many pieces are left to complete the jigsaw.

11. (i) A snow-sled is $0.1\dot{5}$ of the way down a 450 metre hill. Calculate the complementary decimal that represents how far the sled has left to go.

(ii) How far does the sled have left to go to get to the bottom of the hill?

Percentage of an amount

12 When a thunder storm passed over the town of Zappville, the golf-course received $\frac{1}{3}$ of the 78 lightning strikes caused by the storm.

(i) Calculate the complementary percentage that represents the number of strikes that did not occur on the golf course.

(ii) How many lightning strikes were not on the golf course?

13 A survey was posted out to 252 houses in a suburb to see what services residents would like to have improved for their local community. The post office received back $83\frac{1}{3}\%$ of the envelopes mailed out with completed surveys. How many surveys were not returned?

14 $14\frac{2}{7}\%$ of the dots printed on a normal six-sided die are located in the centre of the side. How many other non-centred dots are there on a normal six-sided die?

15 The water level in an 18 metre deep well is $0.795\dot{3}$ of the well's depth from the top.

(i) How far from the top of the well is the water level, accurate to 1 decimal place?

(ii) The well is only ever less than 25% full during drought conditions. Determine if this water level indicates drought conditions or not.

One amount as a percentage of another

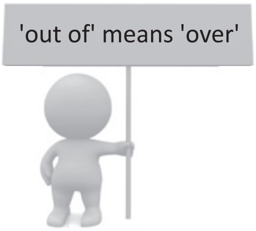
The results of comparing values such as scores out of a total amount are often given as a percentage.

Score of 17 out of 25 $\xrightarrow{\text{Fraction}}$ $= \frac{17}{25}$

either $= \frac{68}{100}$ Equivalent fraction

or $= 0.68$ Equivalent decimal

$= 68\%$ Percentage



$= \frac{17}{25} \times 100$ or $17 \div 25 \times 100$

If using a calculator, here are two ways to do it

Here is where this type of percentage calculation can be applied:

10 seconds have passed on a stop-watch. If the person pushes 'stop' after a further 26 seconds, what percentage of one minute (60 seconds) has passed?

$$10 \text{ seconds} + 26 \text{ seconds} = 36 \text{ seconds}$$

$$\therefore \frac{36}{60} = 60\%$$

Let's look at a few more examples before trying some for yourself.

These examples are more complex and require the use of a calculator

(i) What percentage does $12\frac{1}{5}$ out of 25 represent?

$$12.2 \text{ out of } 25 = \frac{12.2}{25} \times 100\% \quad \text{Equivalent decimal}$$

$$= \frac{1220}{25}\% \quad \text{Multiplied by } 100$$

$$= 48.8\% \text{ or } 48\frac{4}{5}\% \quad \text{Decimal and mixed numeral form}$$

You can also be asked to calculate the complementary percentage.

(ii) Hugo scored 23.5 out of a possible 30 points in a recent competition. What percentage represents the number of points he did not score?

$$\text{Points not scored} = 30 - 23.5$$

$$= 6.5$$

$$\therefore 6.5 \text{ out of } 30 = \frac{6.5}{30} \times 100\%$$

$$= \frac{650}{30}\% \quad \text{Multiplied by } 100$$

$$= 21.\dot{6} \text{ or } 21\frac{2}{3}\% \quad \text{Decimal and mixed numeral form}$$

One amount as a percentage of another

1 Calculate these percentage amounts:

a 35 out of 50

b 3.2 out of 5

2 Calculate these percentage amounts, leaving your answers as mixed numeral percentages:

a 17 out of $22\frac{1}{2}$

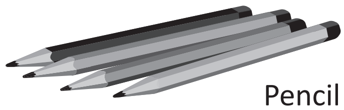
b $\frac{1}{3}$ out of $\frac{4}{5}$

3 Calculate these percentage amounts, rounding your answers to 2 decimal places:

a 10.5 out of 83.4

b $12.2\dot{5}$ out of 50

4 Four different 22.5 gram pieces of lead were studied and found to contain 15.3, 13.05, 18.225 and 16.825 grams of graphite. Calculate the percentage of graphite found in each lead and then use the table below to determine what pencil type each lead was from.



Pencil Type	Graphite	Clay, Wax, etc.
6H	50%	50%
5H	53%	47%
4H	55%	45%
3H	58%	42%
2H	61%	39%
H	63%	37%
HB	68%	32%
2B	73%	27%
3B	75%	25%
4B	78%	22%
5B	81%	19%
6B	84%	16%

a 15.3 grams of graphite — Pencil type

b 13.05 grams of graphite — Pencil type

c 18.225 grams of graphite — Pencil type

d 16.825 grams of graphite — Pencil type

5 Each student in a class of twenty five is asked to select one of forty different vegetables to write a nutrition report about it. The same vegetable cannot be picked twice. What percentage of vegetables do not get a nutrition report written about them?

One amount as a percentage of another

6 It's time for 582 birds to migrate. 368 of the birds start their migration while the remainder of the birds wait until the first flock has left. What percentage of birds leave as part of the second flock? Round your answer to the nearest whole percentage.

7 The surface area of the Earth is roughly $510\,072\,000$ km^2 (or $196\,935\,000$ square miles).

a If $361\,130\,976$ km^2 of the Earth's surface is covered with salt water oceans, what percentage of the Earth's surface is covered in salt water?

b Use the percentage calculated in part a to calculate the surface area of the Earth covered in salt water oceans in square miles (accurate to the nearest whole square mile).

8 A video goes viral and attracts an average of 980.24 hits every second in the first five minutes. After one hour it has had $1\,225\,300$ hits. What percentage of the hits occurred in the first five minutes? Psst: 1 minute $= 60$ seconds

9 If $70\frac{4}{5}$ gallons of rain water has been released from a water tank, what percentage of water remains if the $125\frac{3}{4}$ gallon tank was full before the water was released? Round answer to 2 d.p.

Percentage change

There are two types of percentage changes: increases and decreases.

- An increase means you finish with more than what you started with.

  So we add the percentage amount to the whole amount (100%) that we had to begin with.

  If a javelin thrower beats their personal best distance by 5%, then the new personal best distance is:

$$\underbrace{100\%}_{\text{Initial personal best distance thrown}} + \underbrace{5\%}_{\text{Percentage of extra distance}} = 105\% \text{ of the previous personal best}$$

- A decrease means you finish with a less than what you started with.

  So subtract the percentage from the whole amount (100%) that we had to begin with.

  If a skater reduces their personal best time around a track by 11%, then the new personal best time is:

$$\underbrace{100\%}_{\text{Initial personal best time around track}} - \underbrace{11\%}_{\text{Percentage of time reduced by}} = 89\% \text{ of the previous personal best time}$$

Perform these percentage change calculations

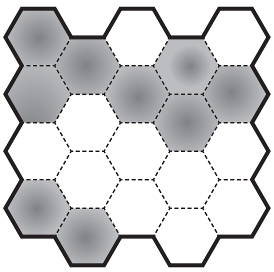
(i) Increase 20 by 15%.

$$\begin{aligned}\text{An increase of } 15\% &= 100\% + 15\% \\ &= 115\% \text{ of original amount} \\ \therefore 115\% \text{ of } 20 &= 1.15 \times 20 \\ &= 23\end{aligned}$$

(ii) Decrease 5.2 by 25%.

$$\begin{aligned}\text{A Decrease of } 25\% &= 100\% - 25\% \\ &= 75\% \text{ of original amount} \\ \therefore 75\% \text{ of } 5.2 &= 0.75 \times 5.2 \\ &= 3.9 \quad \text{or} \quad 3\tfrac{9}{10}\end{aligned}$$

(iii) How many hexagons must be filled to increase the number of shaded hexagons by $33\frac{1}{3}\%$?



$\therefore$ just focusing on the nine shaded hexagons, not all the hexagons.

$$\begin{aligned}\text{An increase of } 33\tfrac{1}{3}\% &= 100\% + 33\tfrac{1}{3}\% \\ &= 133\tfrac{1}{3}\% \text{ of original amount} \\ \therefore 133\tfrac{1}{3}\% \text{ of 9 hexagons} &= 1.\dot{3} \times 9 \text{ hexagons} \\ &= 12 \text{ hexagons}\end{aligned}$$

$\therefore$ 3 more hexagons must be shaded to increase the number of shaded hexagons by $33\frac{1}{3}\%$.

Percentage change

1 Complete these calculations to see the two different ways you can calculate percentage changes.

a Increase 25 by 10%

10% increase $= 100\% \boxed{} 10\%$

$= \boxed{}$ % of initial amount

$\therefore$ Increase of 10% $= \underbrace{\boxed{}}_{\text{Decimal form}} \times 25 = \boxed{}$

Other method:

Find 10% of 25 and add to 25

10% of 25 $= \underbrace{\boxed{}}_{\text{Decimal form}} \times 25$

$= \boxed{}$

$\therefore$ Increase of 10% $= \boxed{} + 25 = \boxed{}$

b Decrease 65 by 40%

40% decrease $= 100\% \boxed{} 40\%$

$= \boxed{}$ % of initial amount

$\therefore$ Decrease of 40% $= \underbrace{\boxed{}}_{\text{Decimal form}} \times 65 = \boxed{}$

Other method:

Find 40% of 65 and subtract from 65

40% of 65 $= \underbrace{\boxed{}}_{\text{Decimal form}} \times 65$

$= \boxed{}$

$\therefore$ Decrease of 40% $= 65 - \boxed{} = \boxed{}$

2 Calculate these percentage changes:

a Decrease 50 by 30%.

b Increase 76 by 25%.

c Increase 6.2 by 5%.

d Reduce 8 by 1.5%.

e Mark 258 larger by $33\frac{1}{3}\%$.

f Add a further 92% to $12\frac{4}{5}\%$.

3 **a** Decrease 50 cars by 100%.

b Can you decrease a physical quantity (such as cars) by more than 100%? Briefly explain your answer.

Percentage change

4 **a** Increase 70 by 100%

b Write down another mathematical calculation which also increases a number by 100%.

c Increase 70 by 200%.

d Write down another mathematical calculation which also increases a number by 200%

e Write down a general rule you could use to make increasing a number by a multiple of 100% quick and simple.

5 **a** Increase $\$20$ by 50%.

b Reduce the answer to part **a** by 50%.

c Did you expect to get the same amount of $\$20$ for part **b** because you were increasing and decreasing by the same percentage?
Explain briefly why you think the answer to part **b** could not be $\$20$.

6 **a** Lengthen 80 m by 20%, then increase this length by a further 25%.

b Is increasing a number by 20% and then increasing the answer by 25% the same as increasing the original amount by 45%? Briefly explain why/why not.

Percentage change

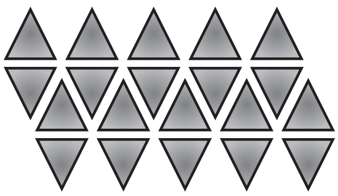
7 a Increase 60 by $33\frac{1}{3}\%$, then decrease the answer by 20%.

b Reduce 200 by 25%, then reduce the result by 70.5%.

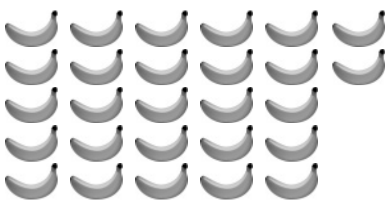
c Decrease $\$55$ by 12%, then increase the result by 16%. Round answer to 2 decimal places.

d Inflate $\$90$ by $5\frac{1}{4}\%$, then reduce the result by $\frac{3}{5}\%$. Round the **final** answer to 2 decimal places.

8 a Increase the number of triangles in this image by 45%.



b Cross out enough bananas below so the total number of bananas has shrunk by $66\frac{2}{3}\%$.



c 60% of the squares below were shaded in before 75% of the shaded squares were cleared again. Calculate how many squares would now be shaded and shade the diagram appropriately.

Percentage change

9 All the ingredients below are used to prepare a particular dish.
- The mass/quantity of each ingredient required is shown next the name of the ingredient.
- The values in square brackets indicate how many people each given amount is used for.

A chef is making the dish for 4 people. Using percentage change calculations, match each ingredient to the correct percentage change and ingredient amount required for 4 people by drawing a straight line between the dots.
Write each letter in the matching number squares below to reveal the dish this chef is making.
Psst: calculate the percentage change for the number of serves and apply this to the ingredients

Ingredients


- 0.75 red onions [serves 3]


- 0.55 g marinated feta [serves 1]

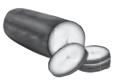

- 20 g Salt [serves 16]


- 7.5 tomatoes [serves 10]


- 120 mL vinegar mix [serves 8]


- 350 g olives [serves 7]


- 187.5 mL olive oil [serves 6]


- $\frac{3}{4}$ of a lebanese cucumber [serves 2]


- $26\frac{1}{4}$ g of oregano [serves 14]


- 225 g chopped red capsicum [serves 5]

viii x v vi ix iv vii iii i ii

Percentage change needed

- 75% decrease
- $71\frac{1}{3}$% decrease
- $66.\dot{6}$% decrease
- 60% reduction
- $33\frac{1}{3}$% increase
- 50% less
- 75% more
- $33.\dot{3}$% decrease
- 300% increase
- 20% removed
- $42\frac{6}{7}$% decrease
- $42\frac{6}{7}$% increase
- 100% increase

L D K S A A E E G R

Quantity needed for 4 people

- 1
- 80 mL
- 3
- 60 mL
- 225 g
- 200 g
- 70 mL
- 7.5 g
- 220 g
- 5 g
- 9.5 g
- 125 mL
- 150 mL
- 180 g
- 1.5
- 2
- 4

									D
(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)

Unitary method

Unitary method for percentages is a mathematician's way of saying "find how much 1% is first".

Once you know what amount 1% represents, you can calculate any other percentage amount.

A 210 g slice of birthday cake is 15% of the whole cake. What is the mass of the birthday cake?

15% of the birthday cake $= 210$ g

$\therefore$ 1% of the birthday cake $= 210$ g $\div 15$ $= 14$ g — Divide by 15% amount by 15 to get 1%

$\therefore$ 100% of the birthday cake $= 14$ g $\times 100$ $= 1400$ g — Multiply 1% amount by 100 to get 100%

$\therefore$ The mass of the birthday cake $= 1400$ g or 1.4 kg — Answer with a satement

Here are some more examples using unitary method for different circumstances

(i) After a recent survey, 27 new saplings of a rare tree were discovered. If this is an increase of 135% from a previous survey, how many saplings of this rare tree were discovered during the previous survey?

135% of the previous survey $= 27$ saplings

$\therefore$ 1% of the previous survey $= 27$ saplings $\div 135$ $= 0.2$ saplings — Divide by 135 to get 1%

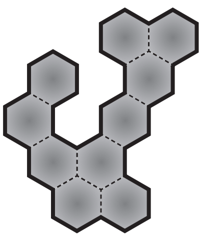
100% of the previous survey $= 0.2$ saplings $\times 100$ — Multiply 1% amount by 100

$\therefore$ 100% of the previous survey $= 20$ saplings

$\therefore$ The saplings discovered in the previous survey $= 20$ — Answer with a statement

(ii) $66\frac{2}{3}\%$ of an image made entirely of equal-sized hexagons is visible below. How many hexagons would you see if 80% of this image was made visible?

The number of hexagons visible $= 10$



$\therefore$ $66\frac{2}{3}\%$ of the full picture $= 10$ hexagons

$\therefore$ 1% of the full picture $= 10$ hexagons $\div 66\frac{2}{3}\%$ $= 0.15$ hexagons — Divide by $66\frac{2}{3}\%$ to get 1%

$\therefore$ 80% of the full picture $= 0.15$ hexagons $\times 80$ $= 12$ hexagons — Multiply by 80 to get 80%

$\therefore$ The number of hexagons visible in 80% of the full image $= 12$ — Answer with a statement

Unitary method

1 Complete these calculations using the unitary method to find 100% of the given amount.

a 25% of the amount is 40.

25% of the amount $= \boxed{}$

$\therefore 1\%$ of the amount $= \boxed{} \div \boxed{}$

$= \boxed{}$

$\therefore 100\%$ of the amount $= \boxed{} \times \boxed{}$

$= \boxed{}$

b 112% of the amount is 84.

112% of the amount $= \boxed{}$

$\therefore 1\%$ of the amount $= \boxed{} \div \boxed{}$

$= \boxed{}$

$\therefore 100\%$ of the amount $= \boxed{} \times \boxed{}$

$= \boxed{}$

2 Calculate the amount represented by 100% for each of these values. Show all your working.

a 55% of the amount is 220.

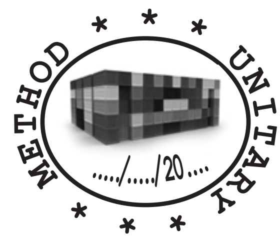
b 325% of the amount is 487.5.

c 34.2% of the amount is 282.15.

d $115\frac{3}{8}\%$ of an amount is $1269\frac{1}{8}$.

e 286% of the amount is $16\,094$.

f $14\frac{2}{3}\%$ of an amount is $7.1\dot{6}$.



Unitary method

3 Calculate the percentage amount for each of these given quantities. Show all your working.

a 65% of a length is 390 units.
How long is 32% of the same length?

b 12% of an ingredient is 46 g.
How much is 30% of the same ingredient?

c 110% of a mass is 5 kg.
What is 44% of the same mass?

d 290% of the initial amoeba population is 4096.
How many amoeba are there in of $72\frac{1}{2}\%$ the initial population?

4 Use the unitary method to solve these problems:

a Concrete is made by mixing cement with sand and gravel. If sand makes up 35% of the whole mixture, what mass of concrete is produced if a 56 kg bag of sand was used?

b In one particular termite nest there are 8575 worker termites. They account for $87\frac{1}{2}\%$ of the entire colony. How many soldier termites are there in the same nest if they account for 11.5% of the colony?

c Two fifths of the $41\,250$ people at a sport venue represent 60% of all the registered fans of the Pesky Pi-rates team. How many people are registered fans of the Pesky Pi-rates?

Unitary method

Here are some more examples using unitary method for different circumstances.

$$\frac{\text{New percentage}}{\text{Initial percentage}} \times \text{Initial percentage amount}$$

(i) 25% of a number is 120. What is the number (ie 100%)?

$$\frac{100}{25} \times 120 = 480$$

(ii) 15% of a number is 45. What is 72% of the same number?

$$\frac{72}{15} \times 45 = 216$$

The calculation of 1% still happens within this formula. Can you see how?

5 Use this rule to calculate these rounding your answers to 2 decimal places.

a 35% of a number is 16. What is the number?

b 450% of a number is 96. What is the amount?

c 71% of a number is 121.
What is 45% of the same number?

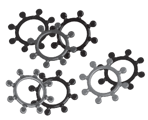
d 128% of a value is 245.8.
What is 50% of the same vlaue?

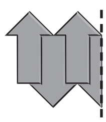
e 30% of a fraction is $\frac{5}{8}$.
What is 140% of the same fraction?

f $5\frac{1}{3}\%$ of a timeframe is 2130 seconds.
How long is $10\frac{3}{7}\%$ of the same timeframe?

6 Only 2.5% of these images made entirely of equal-sized objects are visible. How many objects would you see in each image if 70% of the image was made visible?

a 

b 

c 

Profit and loss

Profits and losses are usually money values relating to the sale or trade of items by retailers.

Some important terms first:

- **cost price:** The price retailers pay for the items they are going to sell.
- **marked price:** The price retailers put on the items they are selling to customers
- **sale price:** The price retailers actually sell the items for.

A profit is what you gain from a sale

Sale price $>$ Cost price

Sale price $-$ Cost price $=$ Positive answer

A loss is what you lose from a sale

Sale price $<$ Cost price

Sale price $-$ Cost price $=$ Negative answer

Here is an example including some new terms important to these types of percentage calculations

To make a profit, shops increase the cost price of items by a percentage. This is called a **markup**.

Marked price $=$ Cost price $+$ The **markup**

(i) A retailer buys some basketballs for $\$22$ each and decides to add a 75% mark-up to sell them.

$\$22$ increased by 75% $= \$22 \times 1.75$

$= \$38.50$

Always answer money questions accurate to 2 decimal places.

$\therefore$ Marked price of a basketball in the shop is $\$38.50$.

To make an item sell quicker, they decrease the cost price by a percentage. This is called a **discount**.

Discount/On-Sale Price $=$ Marked price $-$ the **discount**

(ii) The retailer then discounts the marked price of the basketballs by 12% to sell them faster.

$\$38.50$ decreased by $12\% = \$38.50 \times 0.88$

$= \$33.88$

$\therefore$ Discounted (or "on sale") price of a basketball in the shop is $\$33.88$

(iii) Calculate the profit or loss for the retailer if a basketball was sold at the discounted price.

Sale price $-$ Cost price $= \$33.88 \times \$22$

$= \$11.88$

Positive answer, so the retailer made a profit of $\$11.88$

Profit and loss

1 Calculate the profit or loss for each of these cost and sale prices.

a. Cost price = $\$25$
Sale price = $\$22$
∴ ☐ Profit ☐ Loss

b. Cost price = $\$45.00$
Sale price = $\$63.50$
∴ ☐ Profit ☐ Loss

c. Cost price = $\$104.25$
Sale price = $\$140.75$
∴ ☐ Profit ☐ Loss

2 To calculate the profit or loss as a percentage of the cost price, we use this formula:

Profit/loss as a percentage of the cost price = (profit/loss ÷ cost price) $\times 100\%$

Complete these calculations which follow this formula to find the percentage profit/loss

a. Cost price = $\$18.00$
Sale price = $\$22.50$
☐ Profit ☐ Loss
∴ (☐ ÷ ☐) $\times 100\%$
= ☐ % ☐ Profit of the cost price ☐ Loss of the cost price

b. Cost price = $\$5.20$
Sale price = $\$4.16$
☐ Profit ☐ Loss
∴ (☐ ÷ ☐) $\times 100\%$
= ☐ % ☐ Profit of the cost price ☐ Loss of the cost price

3 To calculate the profit or loss as a percentage of the sale price, we use a similar formula:

Profit/loss as a percentage of the sale price = (profit/loss ÷ sale price) $\times 100\%$

Complete these calculations which follow this formula to find the percentage profit/loss.

a. Cost price = $\$34.56$
Sale price = $\$25.60$
☐ Profit ☐ Loss
∴ (☐ ÷ ☐) $\times 100\%$
≈ ☐ % ☐ Profit of the sale price ☐ Loss of the sale price

b. Cost price = $\$49.66$
Sale price = $\$124.15$
☐ Profit ☐ Loss
∴ (☐ ÷ ☐) $\times 100\%$
≈ ☐ % ☐ Profit of the sale price ☐ Loss of the sale price

4 Calculate the percentage profit/loss (to 1 d.p.) of the cost and sale prices for these:

a. Cost price = $\$20.50$
Sale price = $\$11.45$
☐ Difference ☐ Profit ☐ Loss
= ☐ % ☐ Profit of the cost price ☐ Loss of the cost price
= ☐ % ☐ Profit of the cost price ☐ Loss of the cost price

b. Cost price = $\$59.40$
Sale price = $\$84.95$
☐ Difference ☐ Profit ☐ Loss
= ☐ % ☐ Profit of the cost price ☐ Loss of the cost price
= ☐ % ☐ Profit of the cost price ☐ Loss of the cost price

Profit and loss

5 Calculate the marked price for these items after the given percentage markups.

a Cost price $= \$24$
Markup $= 25\%$

Marked price $= \$$ ______

b Cost price $= \$32.50$
Markup $= 46\%$

Marked price $= \$$ ______

c Cost price $= \$230.00$
Markup $= 12.5\%$

Marked price $= \$$ ______

6 A store displayed the sign below in its window. Calculate the discounted price for these items:

a Marked price $= \$157.00$

Discounted price $= \$$ ______

b Marked price $= \$78.50$

Discounted price $= \$$ ______

7 A store purchases 10 dining tables for $\$210$ per table. The markup put on each table by the store was 50%.

a Five tables were successfully sold for the marked price. Calculate the profit made from the sale of these five tables.

b Write the profit made as a percentage of the cost price for the five tables sold.

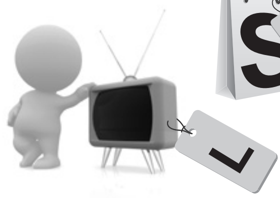
c The store discounted the marked price of the five remaining tables by 40% to make room for new stock. Calculate the new discounted price for each table.

d Calculate the profit/loss made on the sale of all ten tables.

Profit and loss

The unitary method is helpful for these profit and loss questions:

8 A store held a 'special letter tag' sale. Any item in the store labelled with an S, A, L or E tag was discounted by the percentages shown below (-70% means a discount of 70%). Use the given information to calculate the marked price for the items tagged below:



a Television

Discounted price = $\$3150$

Percentage discount = ☐

∴ Marked price = ☐

b Pirate hat

Discounted price = $\$55.80$

Percentage discount = ☐

∴ Marked price = ☐

c Megaphone

Discounted price = $\$97.50$

Percentage discount = ☐

∴ Marked price = ☐

d Golf clubs

Discounted price = $\$455.60$

Percentage discount = ☐

∴ Marked price = ☐

9 An item costing $\$40$ is given a mark up of 20%. If the item was then given a 15% discount when sold, did retailer make a profit or a loss? Show the calculations used to get your answer.

Profit and loss

10 When an item is sold for the same amount as the cost price, it is called 'breaking even' (or the retailer says they 'broke even on the sale').
A set of crystal glasses were bought by a retailer for $\$200$ and given a markup of 35%.

a Calculate the marked price on the crystal glasses in the retail store.

b What percentage discount (to 1 d.p.) was given to a buyer if the retailer broke even on the sale?

11 The cost price of a formal dress was 12.5% more than the sale price of $\$890$.

a How much money did the seller lose on the sale of this dress?

b What was the labelled price of the dress if the cost price was given a markup of 20%?

c Calculate the discount given to this dress at the time of its sale as a percentage of the cost price (to the nearest whole percent).

12 A retailer decides that a 17.25% profit is needed on a new range of products if it is going to continue selling them. The new products were purchased at a cost price of $\$72.50$ each.

a The products are marked up by $36.\dot{3}\%$. What is the marked price for these products to 2 d.p.?

b What is the minimum price (to the nearest whole dollar) the items must be sold for so the retailer will continue selling them?

c Calculate the maximum percentage discount that can be given to the marked price to ensure the retailer continues to sell the product (to nearest whole percent)?

More applications of percentage calculations

1. Look at this nutritional information for a packet of food.

Nutritional Information

Serving size: 5 g

	Avg Qty per Serve	Avg Qty per 100g
Energy	78 kJ	1550 kJ
Protein	0.4 g	8.0 g
Fat, total	0.2 g	4.4 g
-saturated	0.0 g	0.4 g
Carbohydrate	3.8 g	75.0 g
-sugars	1.3 g	25.9 g
Sodium	8 mg	165 mg
Vitamin C	75 mg	1500 mg
Vitamin A	1539 IU	28390 IU

a) Calculate your minimum daily requirement of protein.

Psst: remember, percent mean per 100!!

The standard method used by nutritionists to estimate our minimum daily grams of protein needed is to multiply the body weight in kilograms by 0.8.

b) Use the formula to calculate the mass of protein recommended for an adult who weighs 70 kg.

c) How many servings of this food will give this person their recommended daily source of protein?

d) If a 70 kg adult eats just enough servings to get their required daily source of protein, how many grams of carbohydrates will also be eaten?

e) An expert recommends splitting your total daily food into 50% carbohydrates, 32% proteins and 18% healthy fats for a healthy lifestyle. Use percentage calculations to show whether or not this food is a healthy option by comparing the amount of protein and carbohydrate eaten by this adult.

2. Planet X11 orbits its sun every 145 days. This is 24.6% of the time taken for Planet G23 to orbit the same sun. How long does it take for Planet G23 to complete 2 orbits to the nearest whole day?

Psst: 2 orbits is like 200%!

More applications of percentage calculations

3 Every year, the height of a pine tree in a renewable forestry plantation increases by $20\frac{1}{4}\%$ of the height it reached the year before. The height of one particular tree now is 14 m.

a How high will this tree be one year from now?

b How much further will the grow between one and two years from now to 2 decimal places?

c By what overall percentage will the tree have grown by two years from now to 1 decimal place?

d Once the tree reaches a height of 25 m, it is cut down and used for timber products. Will this tree be cut down three years from now? Show working to back up your answer.

e If allowed to continue growing, four years from now the tree will be more than double the current height. Explain how this is possible if it has only grown by $20\frac{1}{4}$ each year.

4 This one deserves an awesome passport stamp once completed.
While flying 50 m above the ground, a hot air balloon is filled with hot air so its height above the ground increased by 31%. After 20 minutes, the balloon had already lowered 17 m before more hot air is put into the balloon, raising the heigh above the ground at that time by $28.8\dot{3}\%$. Calculate the percentage change in height from the initial 50 m above ground to nearest whole percent.

Here is a summary of some important things to remember for percentage calculations

Percentage of an amount

Calculate 20% of 10 → 'of' means $\times$

$\frac{20}{100} \times 10 = 2$ Using the equivalent fraction

or

$0.2 \times 10 = 2$ Using the equivalent decimal

One amount as a percentage of another

Score of 16 out of 20 $\xrightarrow{\text{Fraction}}$ $= \frac{16}{20} = \left[\frac{16}{20} \times 100\right]$ or $[(16 \div 20) \times 100]\%$ $= 80\%$ percentage

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Complementary percentages

Like complementary fractions.
If 20% of the ingredients is flour, then the rest of the ingredient is $100\% - 20\% = 80\%$.

Percentage change

Percentage increase
Add the percentage amount to 100%

Percentage decrease
Subtract the percentage amount from 100%

Profit and Loss

Profit $=$ what you gain from a sale

Sale price $>$ Cost price

$\therefore$ Sale price $-$ Cost price $=$ Positive answer

Loss $=$ what you lose from a sale

Sale price $<$ Cost price

$\therefore$ Sale price $-$ Cost price $=$ Negative answer

Both can be expressed as percentages of the cost price or sale price.

- **cost price:** The price retailers pay for the items they are going to sell.
- **marked price:** The price retailers put on the items they are selling to customers.
- **sale price:** The price retailers actually sell the items for.
- **mark up:** The percentage value added to the cost price for selling.
- **discount:** The percentage value deducted from the marked price for selling.

Unitary Method

Divide the amount by the percentage value to find the amount 1% represents.
Multiply the amount 1% by the new percentage value required.

Quick method using calculator: $\dfrac{\text{New percentage}}{\text{Initial percentage}} \times \text{Initial percentage amount}$